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Local stiffness-density correlations for polycarbonate structural foams*

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Most studies on the mechanical properties of foams have focused on correlating pointwise, through-thickness elastic modulus and density variations. These local property correlations have then been used to predict the tensile and flexural moduli of foam bars. However, there is some question as to the meaning of a pointwise local density for a cellular material, and the dependence of local modulus on the local density is difficult to measure. This paper is concerned with correlating the density and tensile moduli of structural foams over a 12.7-mm scale. Data obtained from tests on 6.35-mm- and 4-mm-thick polycarbonate foam plaques, molded at nominal density reductions of 5, 15 and 25%, are used to show that the local average tensile modulus of the material correlates linearly with the local average density of the material. © *2000 Kluwer Academic Publishers*

1. Introduction

Thermoplastic rigid, or structural, foam parts are made by a modified injection molding process, in which the volume of resin injected into the mold is smaller than that of the mold cavity. Then, either by the expansion of dissolved gases in the molten plastic or by gases generated by a chemical reaction in a dispersed blowing agent, the resin expands to fill the mold cavity during the molding process. Structural foam parts have complex morphologies consisting of thin "solid" outer skins surrounding porous inner cores. This morphology results in light structural components that have relatively high strength-to-weight ratios. As with most injection molded parts, foam components are typically thin-walled. The trend is toward reducing the wall thickness of parts, typically from 6.35 mm (0.25 in.) to 4 mm (0.157 in.). Because foam molding is done at relatively low injection pressures, it requires less expensive molds than standard injection molding.

A comprehensive review of the stiffness of structural foams is given in Ref. [1], and a good resource for cellular solids is Ref. [2]. Much of the literature on the mechanical properties of foams is concerned with relating the flexural modulus of a rectangular beam to the foam density. It has been suggested that this "flexural modulus" can be simply related to the macroscopic average foam density [3]. Also, a relationship between the *local*, pointwise (through-thickness varying) modulus and the *local*, pointwise (through-thickness varying) density has been used to predict the flexural modulus [4, 5]. However, there is some question as to the meaning of a pointwise local density for a cellular material, and the dependence of local modulus on the local density is difficult to measure.

This approach has a major shortcoming: The "flexural modulus" is not a material property of structural foam because it depends on the geometry of the test specimen. For example, the "flexural modulus" for rectangular and channel-section beams are different. Furthermore, the "flexural modulus" cannot even be used to predict the stiffness of a foam bar in tension. (The "flexural" moduli of foams are known to be *significantly* larger than the "tensile" modulus.) Thus, it is unclear as to what stiffness property is appropriate for predicting the structural response of complex structures to general loads.

The nonhomogeneous cellular morphology of structural foams therefore raises several issues regarding mechanical properties and their use to predict the stiffness and strength of parts: If material properties vary from point to point, then what is measured in a test such as the tensile test? How are such "tensile" properties related to those measured in a flexural test? Even more importantly, how can such properties be used to predict the mechanical performance of parts? Should test specimens be molded to a final shape, or be cut from larger pieces? What tests should be used for determining the mechanical properties of foams? An analytical *continuum* framework, in which the through-thickness heterogeneous cellular morphology of foams is approximated by a through-thickness *nonhomogeneous continuum*, has been developed to answer these questions [6–9]. The Young's modulus of the model material (nonhomogeneous continuum) is assumed to vary through the thickness, so that instead of the single value for the stiffness associated with homogeneous materials, the material stiffness is defined by a function. It is then shown that the "tensile modulus" and

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the "flexural modulus" measured by tensile and bend tests, respectively, are *system properties* that depend on the material, the type of test, and on the specimen geometry [6]. Procedures for obtaining the through-thickness variation of the elastic modulus from measurement of the system tensile and flexural stiffnesses have been outlined [6]. Further, it has been shown that for most applications the system tensile stiffness can be used for evaluating the stiffness of thin-walled parts [9].

The questions of appropriate specimen shapes and tests for determining system mechanical properties are addressed in Refs. [10] and [11]. Because the morphologies of individually molded dog-bone foam specimens are not representative of the morphologies of foam parts, specimens cut from molded plaques are recommended [10]. However, a possible problem with this approach could be inconsistencies in the morphologies of specimens cut from different parts of a plaque, and even differences in specimens cut from the same location in different plaques.

Rigid structural foams are described in terms of a general, density-reduction level due to the foaming process. The level of "normal" density reduction is controlled by the process parameters and is directly related to the amount of material injected into the mold; the actual density reduction will vary over the part. Let the density of the (unfoamed) resin be ρ_0 and that of the foamed material be ρ. Then the percent density reduction is given by $100(\rho_0 - \rho)/\rho_0$. Structural foam parts are normally designed for nominal density reduction levels in the range of 5–25%, although higher density reductions are possible.

The issues of consistency of measured data from different specimens from the same plaque, and plaque-to-plaque consistency, have been addressed in Ref. [11]: Tests on modified polyphenylene oxide foams (NORYL® FN215), which will be referred to as M-PPO foams, showed good plaque-to-plaque consistency. Those tests also showed that although the mechanical properties (such as the modulus) vary along a plaque, they correlate with the *mean* density. Eighteen 19.05 × 152.4 mm (0.75 × 6 in.) specimens were cut from 152.4-mm- (6-in.-) wide by 457.2-mm- (18-in.-) long edge-gated molded plaques, with the flow along the length direction. The *mean* densities of each of the numbered (for location) specimens were determined by mass and volume measurements. Measurements were then made on these specimens for determining the system mechanical properties such as the tensile and flexural moduli and the tensile strength. These tests were done on two plaque thicknesses of 4 and 6.35 mm (0.157 and 0.25 in.), each at two different nominal density reductions of 5 and 15%. The stiffness data from the 5 and 15% density reduction plaques correlated well with the mean density (as measured on 19.05 × 152.4 mm × thickness specimens), and appeared to be insensitive to the specimen thickness (4 and 6.35 mm). However, there appeared to be more scatter in the strength and strain-at-failure data. There can be several reasons for this. First, the random nature of the cellular morphology can cause the observed variations. Second, the mean density based on the entire specimen may not be a good measure of the *local* density and the local mechanical properties.

This paper is concerned with correlating the density and tensile moduli of foams as measured over 12.7 × 12.7 mm (0.5 × 0.5 in.) regions. The data are from tests on 6.35-mm- (0.25-in.-) and 4-mm- (0.157-in.-) thick plaques that were molded at nominal density reductions of 5, 15 and 25%. The material tested is a bisphenol-A polycarbonate foam (LEXAN® FL900), which will be referred to a PC-SF, that contains 5 wt % of chopped glass to promote bubble nucleation.

2. Characterization of the density and the tensile and flexural moduli

In structural foams, the size of bubbles varies across the thickness from very small, microscopic cells near the skin to large bubbles—which can have mean diameters of several millimeters—in the mid-thickness region. In *continuum* models for foams such cellularity gradients are accounted for by allowing the model continuum properties to have through-thickness variations [6–9]. While the spatial variations of bubble sizes in the in-plane (normal to thickness) directions at any depth is much smaller than the through-thickness cellularity gradients, properties measured over distances of several thicknesses, say over a 12.7-mm scale, could be dramatically affected by whether or not large bubbles are present in the region over which measurements are made.

Mechanical properties of polymeric materials are generally determined through tensile tests on flat rectangular bars with cross sections that are 12.7-mm wide and have a thickness that is representative of the application. Because of property gradients, the local strain during a tensile test on a nonhomogeneous material (such as structural foam) could be quite different along the two edges of the specimen. Thus, if an extensometer is used to measure strains along an edge, the resulting tensile moduli would depend on which edge it is attached to. These effects can be quite large for nonhomogeneous materials [12], and raise the issue of what a representative mean modulus is, and how it should be determined [13]. Clearly, a mean modulus based on the mean strain across the (12.7-mm) face would be most appropriate. Because of issues of extensometer slippage, such a face modulus is not necessarily easy to determine [12]. It has been suggested that for such materials the strains be simultaneously measured along both edges of the specimen, resulting in two values, E_L and E_R, for the local modulus. A representative mean modulus could then be $E_A = (E_L + E_R)/2$. This is the approach followed in this paper.

All the data in this paper were obtained from specimens cut from 152.4-mm (6-in.) wide by 457.2-mm (18-in.) long, 6.35- and 4-mm- (0.25- and 0.157-in.-) thick molded PC-SF plaques. The plaques, schematically shown in Fig. 1, were edge gated from the top with the flow direction along the length.

2.1. Test Procedure

Tensile moduli were obtained by tests on long (12.7 × 406.4 mm, 0.5 × 16 in.) rectangular specimens:

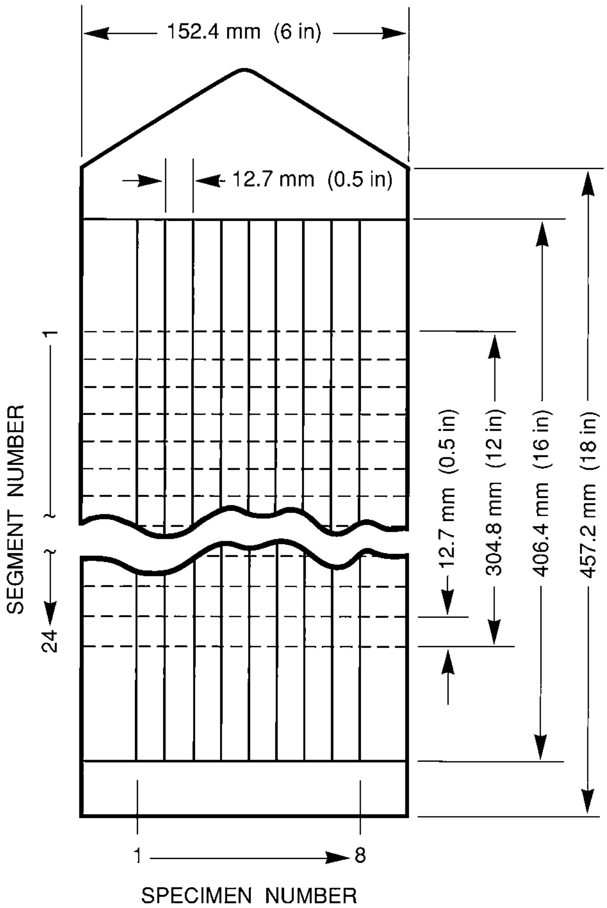


Figure 1 Layout of eight 12.7×406.4-mm specimens cut from 152.4×457.2-mm, edge-gated molded foam plaque.

First, parallel lines were marked with ink on a plaque at 12.7-mm- (0.5-in.-) intervals, as shown by dashed lines in Fig. 1. Then 12.7-mm-wide rectangular specimens (shown by solid lines) were cut, resulting in eight 12.7×406.4 mm rectangular specimens on which twenty-four 12.7-mm-long segments are delineated by parallel ink lines. For each specimen the twenty-four segments were numbered consecutively, with segment 1 being closest to the gated edge (numbering system shown in Fig. 1). The mean cross-sectional area of each segment was calculated by determining the mean width and thickness of each individual segment.

The tensile moduli were determined at each 12.7-mm segment by following the procedure developed for characterizing random glass mat composites [12]. After attaching a 12.7-mm gauge-length extensometer to the left edge of segment 1, the specimen was pulled in tension to a strain of 0.25% a strain rate of 0.01 s^{-1}. The load-strain data were then used to calculate the local left tensile modulus E_L. The specimen was unloaded, and the procedure repeated for each segment. By attaching the extensometer to the right edge of the specimen, the same procedure was then used to determine the right tensile modulus E_R for each segment. In this way, the left modulus E_L and the right modulus E_R were determined at each 12.7×12.7-mm segment over a 101.6×304.8-mm area of the plaque.

After determining the tensile moduli, 3-point bend tests were done on the same set of specimens to determine the mean flexural moduli, over 76.2-mm (3-in.) spans, at three points along each specimen [10]. In these tests, 76.2-mm long spans of the 12.7-mm-wide specimens were centered at the 12.7-mm mark separating two predetermined adjacent segments—the 4–5, 12–13, and 20–21 segment interfaces on each of the eight specimens from each plaque. For the 6.35-mm- and 4-mm-thick specimens, the central load was applied at displacement rates of 1.52 $mm \cdot s^{-1}$ and 2.44 $mm \cdot s^{-1}$, respectively, resulting in a nominal strain rate of 0.01 s^{-1} in the outermost layers. Also, to deform the outermost layers to a nominal strain of 0.25%, central displacements of 0.38 mm and 0.61 mm were used, respectively, for the 6.35-mm- and 4-mm-thick specimens. In this way, in the 3-point bend tests, the strains and strain rates in the outermost layers were nominally the same as in the tensile tests. The flexural moduli at the 4–5, 12–13, and 20–21 segment interfaces were then calculated by using the beam deflection formula for a homogeneous beam.

Finally, after determining the elastic moduli, the specimen was cut along the inked lines resulting in 12.7×12.7-mm coupons, the densities of which were then determined by mass and volume measurements. This procedure was repeated for each of the 24 specimens. In this way, the local density ρ was determined at each 12.7×12.7-mm segment over a 101.6×304.8-mm area of the plaque. Note that, in contrast to the average density—as measured over a $19.05 \times 152.4 \times$ thickness specimen—used in earlier work [10, 11], the local density in this paper is defined over a 12.7×12.7 mm $\times$ thickness (6.35 or 4 mm) volume.

3. Density variations

While the detailed variations of the local density over a 101.6×304.8-mm area of the plaque will be discussed in the following section, the minimum and maximum local densities over this region for 6.35-mm-thick plaques (for nominal density reductions of 5, 15, and 25%) and for 4-mm-thick plaques (for nominal density reductions of 5 and 15%) are listed in Table I. In this table, the numbers in parentheses in columns 4 and 5 are local percent density reductions as calculated from $100(\rho_0 - \rho)/\rho_0$ using an unfoamed (solid) density of $\rho_0 = 1.21$ $g \cdot cm^{-3}$ for the 5% glass-filled PC.

Clearly, the local density reduction can be quite different from the nominal density reduction. For example, in the 6.35-mm-thick plaques, for nominal density reductions of 5, 15, and 25%, the actual density reduction

TABLE I Minimum and maximum local densities in 6.35- and 4-mm-thick polycarbonate structural foam plaques for different nominal density reductions

Specimen thickness (mm)	Nominal density reduction (%)	Local density($g \cdot cm^{-3}$) (Local Density Reduction %)			
		Minimum		Maximum	
6.35	5	1.11	(8.3)	1.17	(3.3)
6.35	15	0.94	(22.3)	1.10	(9.1)
6.35	25	0.88	(27.3)	1.04	(14.0)
4	5	0.97	(19.8)	1.18	(2.5)
4	15	0.88	(27.3)	1.15	(5.0)

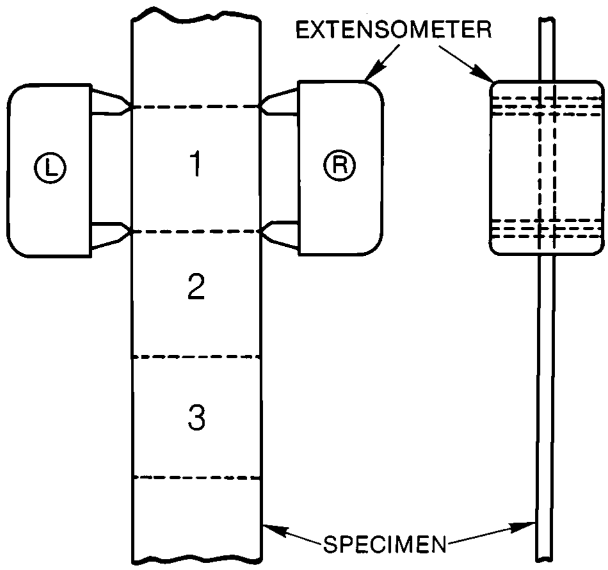


Figure 2 Tensile test extensometer configurations for determining the left and right moduli E_L and E_R along a strip.

varies in the range 3.3–8.3, 9.1–22.3, and 14.0–27.3%, respectively. Thus, plaques with nominal density reductions of 15 and 25% have regions with common local density reductions. Specimen thickness also has an effect. For example, for a nominal density reduction of 5%, while the actual local density reduction varies in the narrow range of 3.3–8.3% for the 6.35-mm-thick specimens, the 4-mm-thick specimens have a much larger variation in the range of 2.5–19.8%. Also, while 6.35-mm-thick plaques with 5 and 15% nominal density reduction do not have regions with the same density reduction, the 4-mm-thick plaques do.

The variations of the local density over 101.6 × 304.8-mm regions of the 6.35-mm-thick plaques (for nominal density reductions of 5, 15, and 25%) and 4-mm-thick plaques (for nominal density reductions of 5 and 15%) are shown in Fig. 3. Clearly, in all cases, the density is the highest at the top (closest to the gate) and drops off with increasing distance from the gate. The variations in the width direction are much smaller. Fig. 3a shows that the 5% density reduction 6.35-mm-thick plaque has a relatively small density variation. A comparison of Fig. 3b and c show that these two plaques with nominal density reductions of 15 and 25%, respectively, have regions with the same density. Also, comparisons of Fig. 3a with d and Fig. 3b with e show that, for the same nominal density reduction, the 6.35- and 4-mm-thick plaques have different local density distributions.

The density variations in a plaque correspond to variations in morphology. Variations in the morphology of a 6.35-mm-thick plaque with a nominal density reduction of 15% are shown in Fig. 4a–f. Fig. 4a, c, and e show the morphologies along the plaque length direction—the morphologies along the sides of the long specimens shown by solid lines in Fig. 1. Fig. 4b, d, and f show the morphologies along the width—the morphologies as seen on sections cut along the dashed lines in Fig. 1. The sections shown in Fig. 4a, b, and c are along the middle of the plaque, i.e., along the common interface between specimen numbers 4 and 5. Fig. 4a and b show the morphologies of a specimen cut from near the gated end, Fig. 4c and d correspond to a specimen from near the middle of the plaque, and Fig. 4e and f correspond to a specimen from near the far end of the plaque. To obtain surfaces that would clearly show the morphologies, lines were scribed on the surface of the plaque and the plaque was then bent to crack it open along the scribed lines. Note that in Fig. 4a the gated end is on the left, so that during filling the flow occurs from the left to the right. In Fig. 4c and e the gated end is on the right, so that flow occurs from the right to the left.

The effect of the flow direction is evident in Fig. 4a, c, and e—the shear stresses in the "parabolic" flow front distort the shape of the bubbles. In keeping with the density gradient along the plaque length, these three figures show totally different morphologies near the gate, in the middle of the plaque, and near its far end. Such bubble distortions are not evident in the transverse sections shown in Fig. 4b, d, and f. At a given point along the length, the morphology along a transverse section is

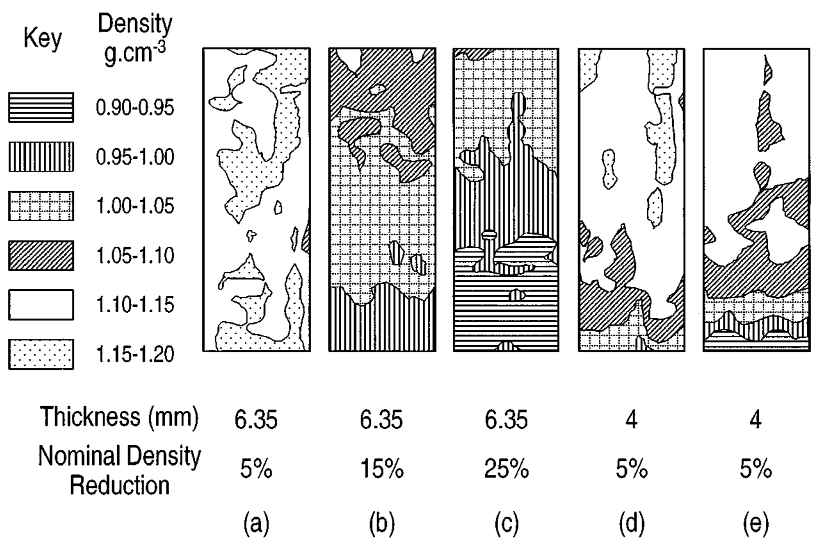
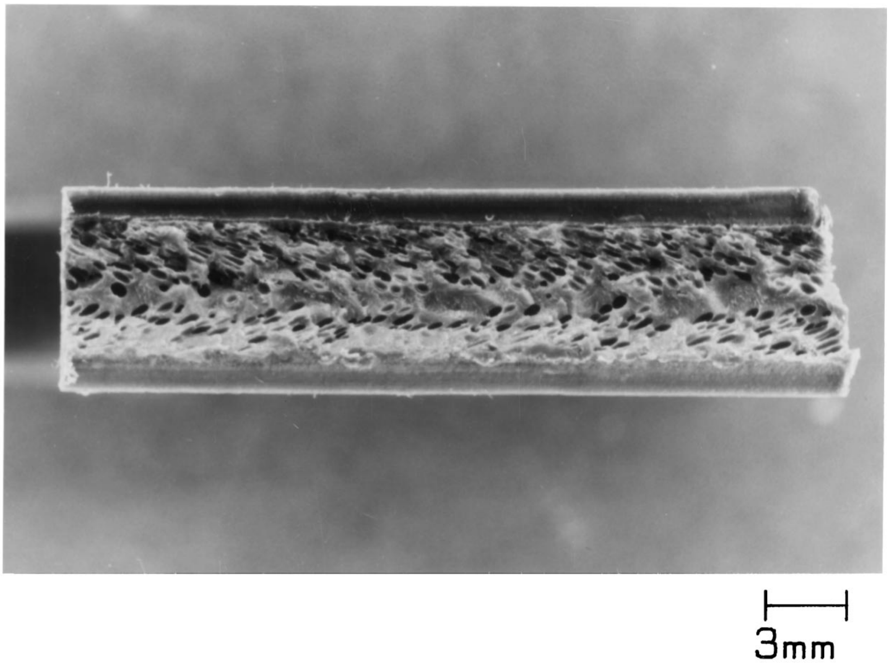
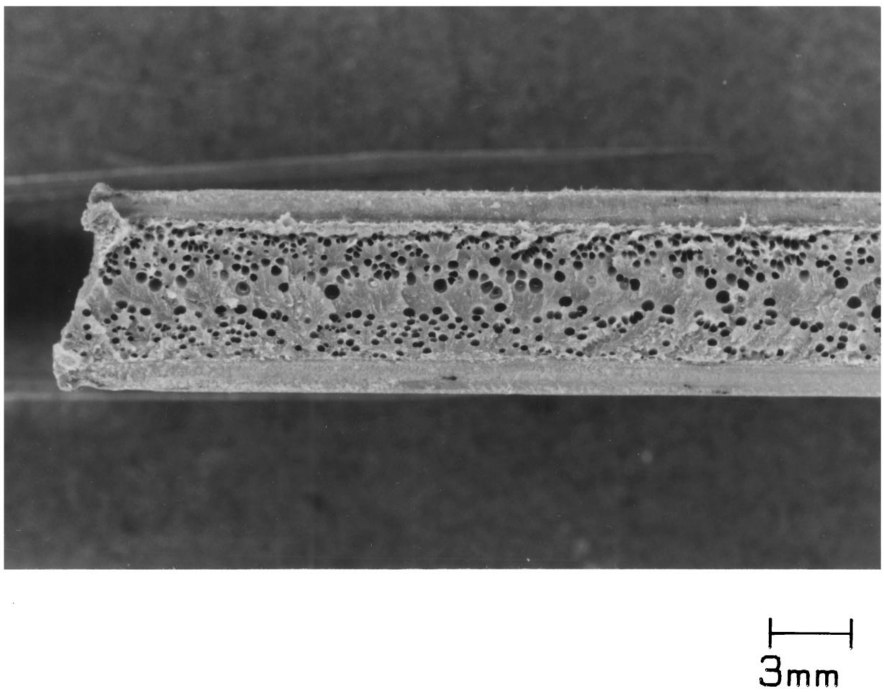


Figure 3 Density reduction contours for 101.6 × 304.8-mm regions of 6.35-mm-thick plaques for nominal density reductions of 5, 15, and 25%, and for 4-mm-thick plaques for nominal density reductions of 5 and 15%.



(a)

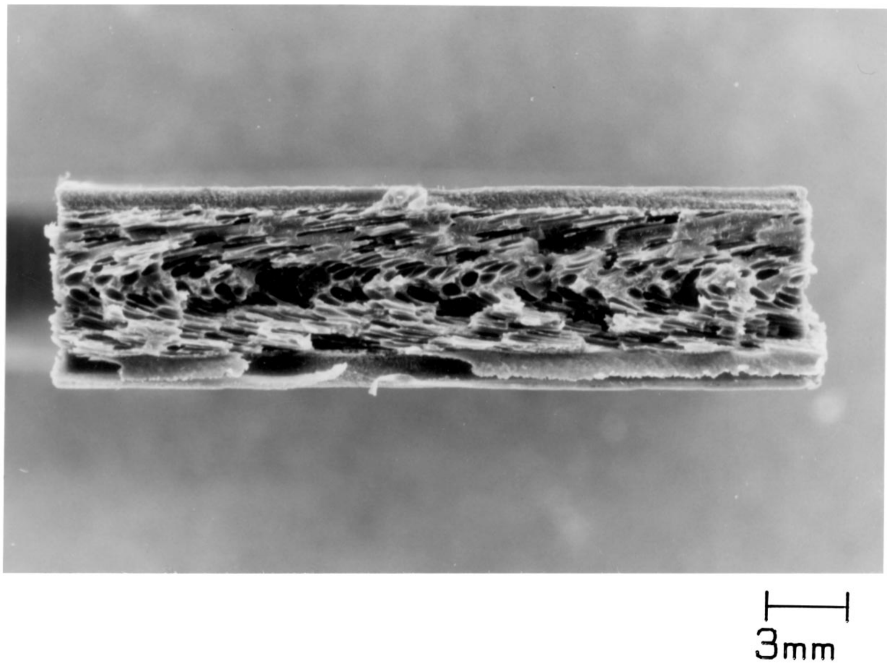


(b)

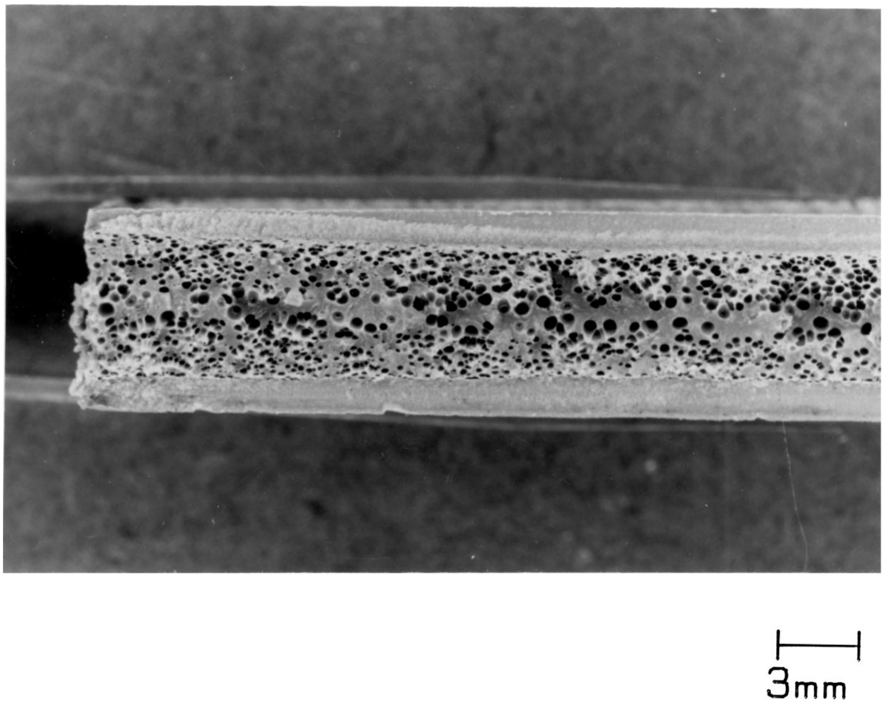
Figure 4 Variations in the morphology of a 6.35-mm-thick plaque with a nominal density reduction of 15%. Fig. 4a, c, and e show the morphologies along the plaque length direction. Fig. 4b, d, and f show the morphologies along the width—as seen on sections cut along the dashed lines in Fig. 1. The sections in Fig. 4a, c, and e are along the middle of the plaque. Fig. 4a and b show the morphologies of a specimen cut from near the gated end, Fig. 4c and d corresponds to a specimen from near the middle of the plaque, and Fig. 4e and f corresponds to a specimen from near the far end of the plaque. Note that in Fig. 4a the gated end is on the left, so that flow occurs from the left to the right. In Fig. 4c and e the gated end is on the right, so that flow occurs from the right to the left. (*Continued*)

relatively homogeneous, corresponding to smaller density variations along in the transverse direction. The morphologies shown in these figures are consistent with the density being the highest near the gate and lowest at the far end. The marked differences in the morphologies in the flow and cross-flow directions should result in anisotropic mechanical properties.

Thus, in a structural foam part, the local density can be very different from that corresponding to the specified nominal density reduction, and may have significant variations across the part. Also, even in parts of the same thickness, the local density at some location in a part with one nominal density reduction may be the same as at some other location in a second part with a different nominal density reduction. Furthermore, the local density can depend on part thickness. Since the local density is known to affect the local mechanical properties of the material, these density variations raise



(c)



(d)

Figure 4 (*Continued*).

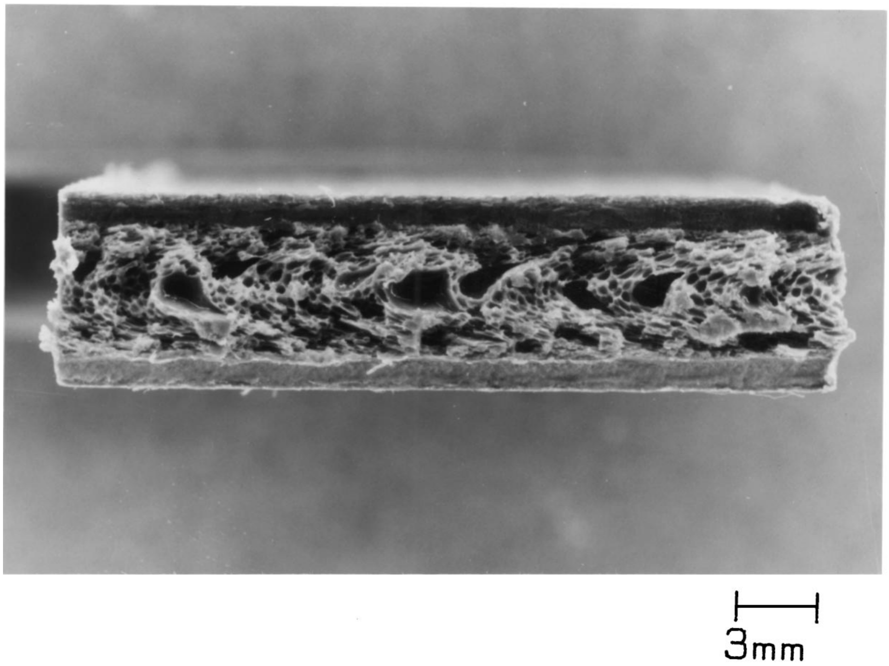
the following important questions: For parts of the same thickness, but of different nominal density reductions, are the mechanical properties of regions having the same local densities the same? That is, for parts of the same thickness, do the local mechanical properties correlate with the local density? Are the local properties of regions of equal local density in parts with different thickness correlated? Such correlations are discussed in the next section.

4. Density and modulus variation correlations

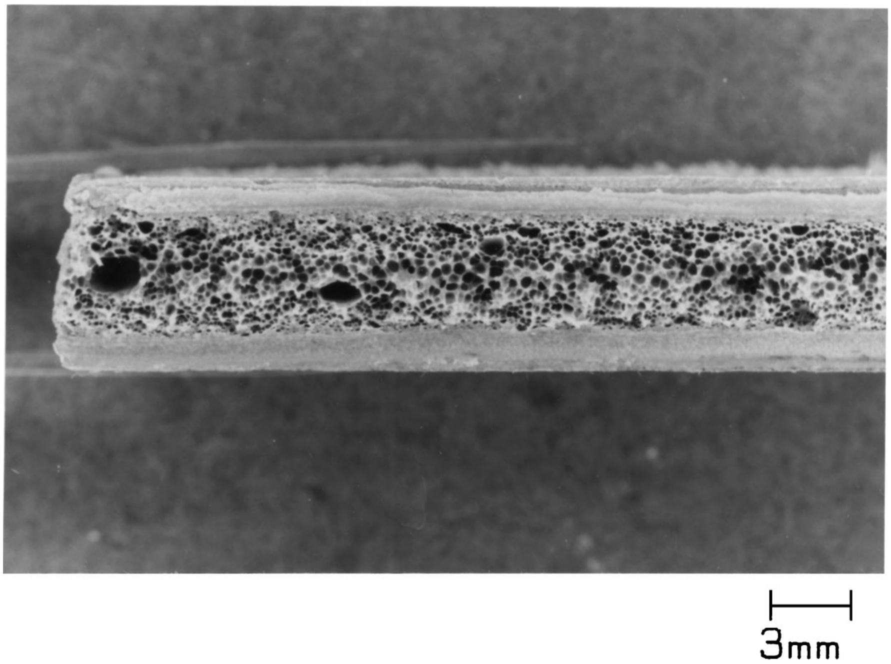
This section is concerned with correlating tensile modulus and density data as measured on a 12.7-mm scale over a 101.6 × 304.8-mm area of 6.35-mm-thick plaques—for three nominal density reductions of 5, 15, and 25%—and 4-mm-thick plaques, for nominal density reductions of 5 and 15%.

4.1. Tensile modulus and density data for 6.35-mm-thick, 5% density reduction foam

The values of the local tensile moduli E_L and E_R, as measured over 12.7 × 12.7-mm regions of one 6.35-mm-thick, 5%-density-reduction plaque, are listed in Table II. In this table, the minimum and maximum values are highlighted in bold. The left modulus E_L varies



(e)



(f)

Figure 4 (*Continued*).

from a minimum of 1.83 GPa (specimen 7, segment 1) to a maximum of 3.42 GPa (specimen 2, segment 24), resulting in a ratio of maximum to minimum of 1.87. The corresponding numbers for the right modulus E_R are 2.10 GPa (specimen 6, segment 1), 3.24 GPa (specimen 3, segment 24), and 1.54, respectively. For each segment, the data for E_L and E_R have been used to calculate the average modulus $E_A = (E_L + E_R)/2$ listed in Table III. The minimum and maximum values of E_A and the ratio of the maximum to the minimum for this plaque are 2.35 GPa (specimen 7, segment 24), 2.93 GPa (specimen 2, segment 24) and 1.27, respectively.

The values of the local density ρ are also listed in Table III. It varies from a minimum of 1.11 $g \cdot cm^{-3}$ (specimen 8, segment 16) to a maximum of 1.17 $g \cdot cm^{-3}$ (in nine segments), the ratio of the maximum to the minimum being 1.05. The fluctuations in the density are caused by differences in the local density reduction, or degree of cellularity.

The variations of E_L, E_R, E_A, and ρ along specimens 1, 4, 5 and 8 are shown in Fig. 5. Clearly, E_L and E_R exhibit significant variations across the plaque. Also, the values of E_L and E_R at a segment can be quite different. However, the variations in E_A are much smaller. By and large, the variation of E_A along each specimen appears to track the variation in density.

Because the local density is a measure of the local cellularity, higher densities should correspond to higher elastic moduli. Fig. 6a–c are, respectively, plots of the

TABLE II Variation of the left and right flow-direction tensile moduli over a 6.35-mm-thick, 5%-density-reduction polycarbonate structural foam plaque

	Tensile modulus (GPa)															
	Specimen number															
Segment number	1		2		3		4		5		6		7		8	
	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R
1	2.71	2.47	2.57	2.55	2.69	2.71	2.45	2.65	2.84	2.55	2.78	**2.10**	**1.83**	2.95	2.84	2.51
2	2.81	2.32	2.69	2.49	2.55	2.68	2.45	2.65	2.60	2.51	2.81	2.33	2.14	3.07	2.52	2.67
3	2.89	2.33	2.82	2.47	2.53	2.75	2.54	2.74	2.55	2.55	2.88	2.40	2.32	2.98	2.60	2.74
4	2.94	2.34	2.76	2.41	2.66	2.55	2.57	2.72	2.58	2.55	2.87	2.52	2.43	2.98	2.45	2.76
5	3.05	2.32	2.72	2.33	2.62	2.69	2.57	2.68	2.46	2.55	2.91	2.54	2.44	2.68	2.43	2.63
6	3.05	2.34	2.87	2.58	2.56	2.71	2.59	2.72	2.44	2.66	2.85	2.54	2.53	2.80	2.52	2.48
7	2.98	2.31	2.74	2.52	2.74	2.71	2.52	2.65	2.49	2.78	2.89	2.32	2.53	2.64	2.52	2.52
8	2.89	2.38	2.79	2.35	2.70	2.74	2.57	2.77	2.49	2.75	2.87	2.36	2.55	2.73	2.52	2.63
9	2.77	2.48	2.76	2.48	2.52	2.70	2.57	2.75	2.55	2.58	2.87	2.54	2.65	2.61	2.57	2.41
10	2.74	2.52	2.78	2.29	2.49	2.86	2.66	2.91	2.55	2.73	2.87	2.56	2.64	2.66	2.64	2.64
11	2.66	2.69	2.80	2.75	2.60	2.91	2.68	2.70	2.60	2.80	2.80	2.49	2.69	2.49	2.75	2.55
12	2.66	2.73	2.68	2.41	2.57	2.86	2.75	2.86	2.60	2.78	2.86	2.57	2.48	2.67	2.86	2.55
13	2.55	2.64	2.70	2.52	2.50	2.77	2.75	2.70	2.58	2.89	2.84	2.59	2.48	2.63	2.75	2.68
14	2.56	2.71	2.63	2.61	2.68	2.72	2.77	2.79	2.65	2.76	2.84	2.53	2.45	2.67	2.71	2.54
15	2.58	2.69	2.59	2.61	2.72	2.76	2.81	2.77	2.59	2.66	2.82	2.52	2.54	2.70	2.61	2.74
16	2.72	2.48	2.53	2.70	2.84	2.77	2.80	2.66	2.66	2.55	2.77	2.42	2.48	2.61	2.63	2.74
17	2.76	2.59	2.53	2.70	2.75	2.79	2.77	2.75	2.61	2.52	2.73	2.42	2.54	2.50	2.64	2.97
18	2.80	2.56	2.51	2.88	2.72	2.75	2.71	2.67	2.70	2.54	2.74	2.45	2.60	2.65	2.67	2.87
19	2.77	2.64	2.54	2.72	2.77	2.68	2.76	2.56	2.74	2.65	2.69	2.87	2.65	2.78	2.58	2.82
20	2.93	2.45	2.59	2.64	2.68	2.86	2.84	2.56	2.76	2.43	2.71	2.82	2.76	2.54	2.68	2.77
21	2.74	2.52	2.65	2.45	2.51	2.99	2.88	2.51	2.88	2.31	2.69	2.87	2.43	2.87	2.75	2.66
22	2.66	2.53	2.84	2.33	2.52	3.11	2.58	2.49	2.97	2.22	2.67	2.87	2.75	2.80	2.78	2.63
23	2.62	2.64	3.08	2.53	2.48	3.14	2.63	2.41	3.01	2.21	2.69	2.93	2.31	2.62	3.05	2.61
24	2.76	2.37	**3.42**	2.45	2.42	**3.24**	2.71	2.32	3.20	2.17	2.82	2.82	2.38	2.32	3.07	2.63

$(E_L)_{min} = 1.83$ GPa, $(E_L)_{max} = 3.42$ GPa, $(E_R)_{min} = 2.10$ GPa, $(E_R)_{max} = 3.24$ GPa.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

TABLE III Variation of the average flow-direction tensile modulus E_A and the local density ρ over a 6.35-mm-thick, 5%-density-reduction polycarbonate structural foam plaque

	Average tensile modulus (GPa) and local density ($g \cdot cm^{-3}$)															
	Specimen number															
Segment number	1		2		3		4		5		6		7		8	
	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ
1	2.59	1.15	2.56	1.15	2.70	1.16	2.55	1.14	2.70	1.16	2.44	1.15	2.39	1.15	2.68	1.17
2	2.56	1.15	2.59	1.14	2.62	1.15	2.55	1.15	2.55	1.13	2.57	1.15	2.60	1.16	2.59	1.15
3	2.61	1.14	2.65	1.15	2.64	1.16	2.64	1.15	2.55	1.16	2.64	1.17	2.65	1.17	2.67	1.15
4	2.64	1.15	2.59	1.15	2.61	1.15	2.65	1.15	2.56	1.15	2.70	1.17	2.71	1.16	2.60	1.15
5	2.69	1.15	2.53	1.17	2.65	1.15	2.63	1.15	2.51	1.15	2.72	1.16	2.56	1.15	2.53	1.14
6	2.69	1.14	2.72	1.14	2.63	1.15	2.66	1.14	2.55	1.15	2.69	1.16	2.66	1.16	2.50	1.14
7	2.65	1.14	2.63	1.15	2.72	1.16	2.59	1.15	2.64	1.15	2.60	1.16	2.59	1.16	2.52	1.13
8	2.63	1.14	2.57	1.15	2.72	1.15	2.67	1.16	2.62	1.15	2.61	1.16	2.64	1.16	2.58	1.14
9	2.63	1.14	2.62	1.16	2.61	1.15	2.66	1.16	2.56	1.15	2.70	1.16	2.63	1.15	2.49	1.13
10	2.63	1.12	2.54	1.15	2.67	1.16	2.79	1.16	2.64	1.15	2.71	1.16	2.65	1.14	2.64	1.13
11	2.67	1.14	2.77	1.15	2.76	1.16	2.69	1.16	2.70	1.16	2.65	1.15	2.59	1.13	2.65	1.13
12	2.70	1.13	2.54	1.15	2.71	1.15	2.80	1.17	2.69	1.15	2.71	1.15	2.58	1.15	2.70	1.13
13	2.60	1.14	2.61	1.15	2.64	1.16	2.72	1.16	2.73	1.15	2.71	1.16	2.55	1.15	2.71	1.12
14	2.64	1.14	2.62	1.13	2.70	1.16	2.78	1.14	2.71	1.15	2.68	1.15	2.56	1.14	2.63	1.13
15	2.64	1.13	2.60	1.14	2.74	1.13	2.79	1.16	2.63	1.15	2.67	1.14	2.62	1.16	2.68	1.12
16	2.60	1.16	2.61	1.15	2.81	1.14	2.73	1.15	2.61	1.15	2.60	1.14	2.54	1.16	2.69	1.11
17	2.67	1.15	2.61	1.15	2.77	1.14	2.76	1.16	2.57	1.14	2.58	1.15	2.52	1.15	2.80	1.14
18	2.68	1.15	2.70	1.15	2.74	1.16	2.69	1.16	2.62	1.15	2.59	1.14	2.63	1.16	2.77	1.15
19	2.70	1.14	2.63	1.15	2.73	1.13	2.66	1.13	2.69	1.15	2.78	1.15	2.71	1.17	2.70	1.14
20	2.69	1.15	2.61	1.14	2.77	1.16	2.70	1.16	2.59	1.15	2.77	1.15	2.65	1.17	2.72	1.13
21	2.63	1.14	2.55	1.15	2.75	1.14	2.70	1.16	2.60	1.15	2.78	1.15	2.65	1.17	2.70	1.13
22	2.60	1.13	2.58	1.15	2.82	1.15	2.53	1.15	2.60	1.15	2.77	1.14	2.77	1.16	2.70	1.14
23	2.63	1.13	2.80	1.16	2.81	1.16	2.52	1.15	2.61	1.15	2.81	1.15	2.47	1.16	2.83	1.14
24	2.57	1.13	**2.93**	1.16	2.83	1.16	2.52	1.15	2.69	1.16	2.82	1.15	**2.35**	1.16	2.85	1.14

$(E_A)_{min} = 2.35$ GPa, $(E_A)_{max} = 2.93$ GPa, $(\rho)_{min} = 1.11\ g \cdot cm^{-3}$, $(\rho)_{max} = 1.17\ g \cdot cm^{-3}$.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

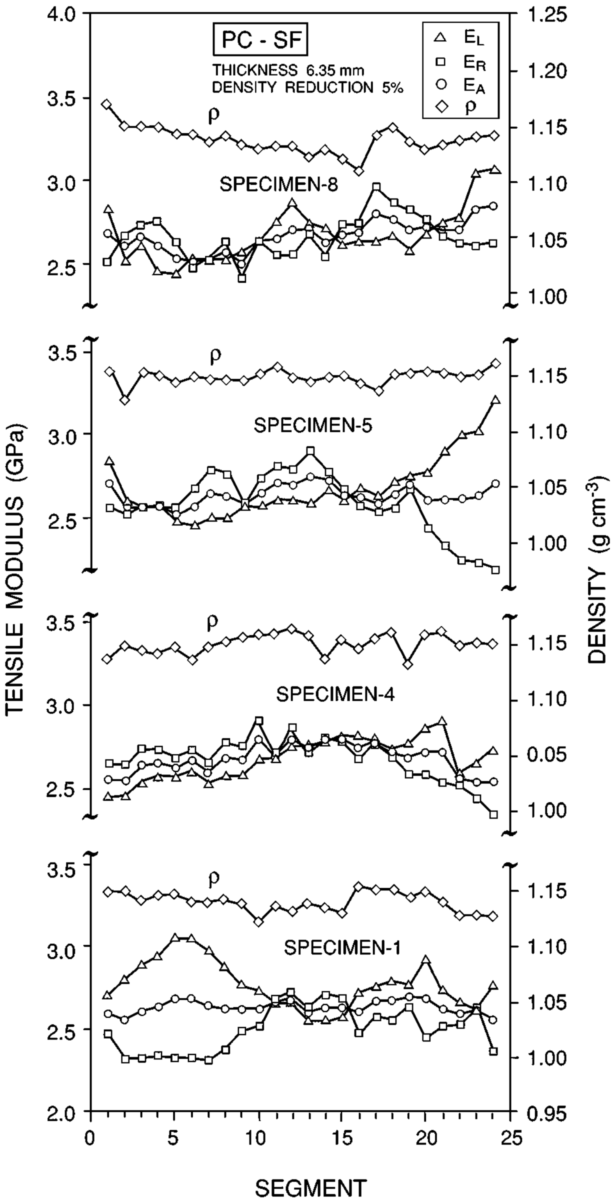


Figure 5 Variations of the Young's moduli E_L, E_R, and E_A and the density ρ at 12.7-mm intervals along specimens 1, 4, 5, and 8 from a 6.35-mm-thick, 5%-density-reduction foam plaque.

local left, right, and average tensile moduli (E_L, E_R, and E_A) versus the local density ρ for the 192 (12.7 × 12.7 mm) segments (data from Tables II and III). Except for isolated points, the densities of most segments lie in a narrow band of about 1.1 to 1.15 $\text{g} \cdot \text{cm}^{-3}$. Fig. 6a and b show that E_L and E_R appear to have a much wider range of variation, which could be interpreted as evidence of a lack of correlation with ρ. However, the fluctuations in E_L are compensated by those in E_R. This can be seen from the plot of E_A versus ρ (Fig. 6c). Most of the values of E_A and ρ are clustered around narrow bands. This figure shows that E_A and ρ are quite strongly correlated. For any given ρ, the values of E_A can be expected to vary depending on the location and orientation of the bubbles. Thus, the apparent scatter in the data can mainly be ascribed to local nonhomogeneities.

The solid line through the data in Fig. 6c is the line $E = 2\rho + 0.35$, in the same units as in the figure. The significance of this line is discussed in Section 4.4. In addition to the tensile modulus, Fig. 6c also displays the nondimensional density (top) ρ/ρ_0—where $\rho_0 = 1.21\ \text{g} \cdot \text{cm}^{-3}$ is the density of unfoamed 5% glass-filled PC—and the nondimensional average tensile modulus (right side) E_A/E_0—where $E_0 = 2.77$ MPa has been chosen such that $E_A/E_0 = 1$ at $\rho/\rho_0 = 1$. Note that $1 - (\rho/\rho_0)$ is the density reduction, so that the nondimensional density scale at the top of this figure provides a direct measure of the local density reduction.

4.2. Tensile modulus and density data for 6.35-mm-thick, 15% density reduction foam

The local tensile moduli E_L and E_R measured over 12.7 × 12.7-mm regions of one 15%-density-reduction plaque are listed in Table IV. The left modulus E_L varies from a minimum of $E_L = 2.00$ GPa (specimen 8, segment 22) to a maximum of 3.20 GPa (specimen 5, segment 1), resulting in a ratio of maximum to minimum of 1.6. The corresponding numbers for the right modulus E_R are 1.64 GPa (specimen 4, segment 23), 2.87 GPa (specimen 8, segment 1), and 1.75, respectively. And the corresponding numbers for E_A (data listed in Table V) are 2.08 GPa (specimen 8, segment 24), 2.70 GPa (specimen 1, segment 2) and 1.30, respectively.

The local minimum and maximum densities for this plaque (data listed in Table V) are 0.94 (in five segments) and 1.10 $\text{g} \cdot \text{cm}^{-3}$ (in four segments). This is a much larger variation than that in the 5%-density-reduction plaque (1.11 to 1.17 $\text{g} \cdot \text{cm}^{-3}$). Also, the local densities in these two plaques do not overlap.

Just as in the case of the 5%-density-reduction plaque (Fig. 6a and b), the data in Table IV show that the left and right tensile moduli for the 15%-density-reduction plaque can be very different at each location, and these two moduli do not correlate well with the local density. However, as can be seen from Fig. 7, the average modulus (data in Table V) correlates fairly well with the local density. Here again, the solid line corresponds to $E = 2\rho + 0.35$. Also, the nondimensional scales in this figure are the same as in Fig. 6c.

4.3. Tensile modulus and density data for 6.35-mm-thick, 25% density reduction foam

Tensile modulus and density data for a 25%-density-reduction foam plaque are listed in Tables VI and VII. The minimum and maximum values, and the ratio of the maximum to the minimum values are: 1.94 GPa (specimen 5, segment 22), 3.84 GPa (specimen 3, segment 1), and 1.98, respectively, for E_L; and 1.55 GPa (specimen 5, segment 1; and specimen 6, segment 24), 2.75 GPa (specimen 8, segment 13), and 1.77, respectively, for E_R. The corresponding numbers for E_A are 2.04 GPa (specimen 3, segment 21), 2.71 GPa (specimen 3, segment 1), and 1.33, respectively.

The values of the local density are listed in Table VII. The density varies from a minimum of 0.88 $\text{g} \cdot \text{cm}^{-3}$ (specimen 5, segment 22; specimen 7, segment 21) to a maximum of 1.04 $\text{g} \cdot \text{cm}^{-3}$ at several locations

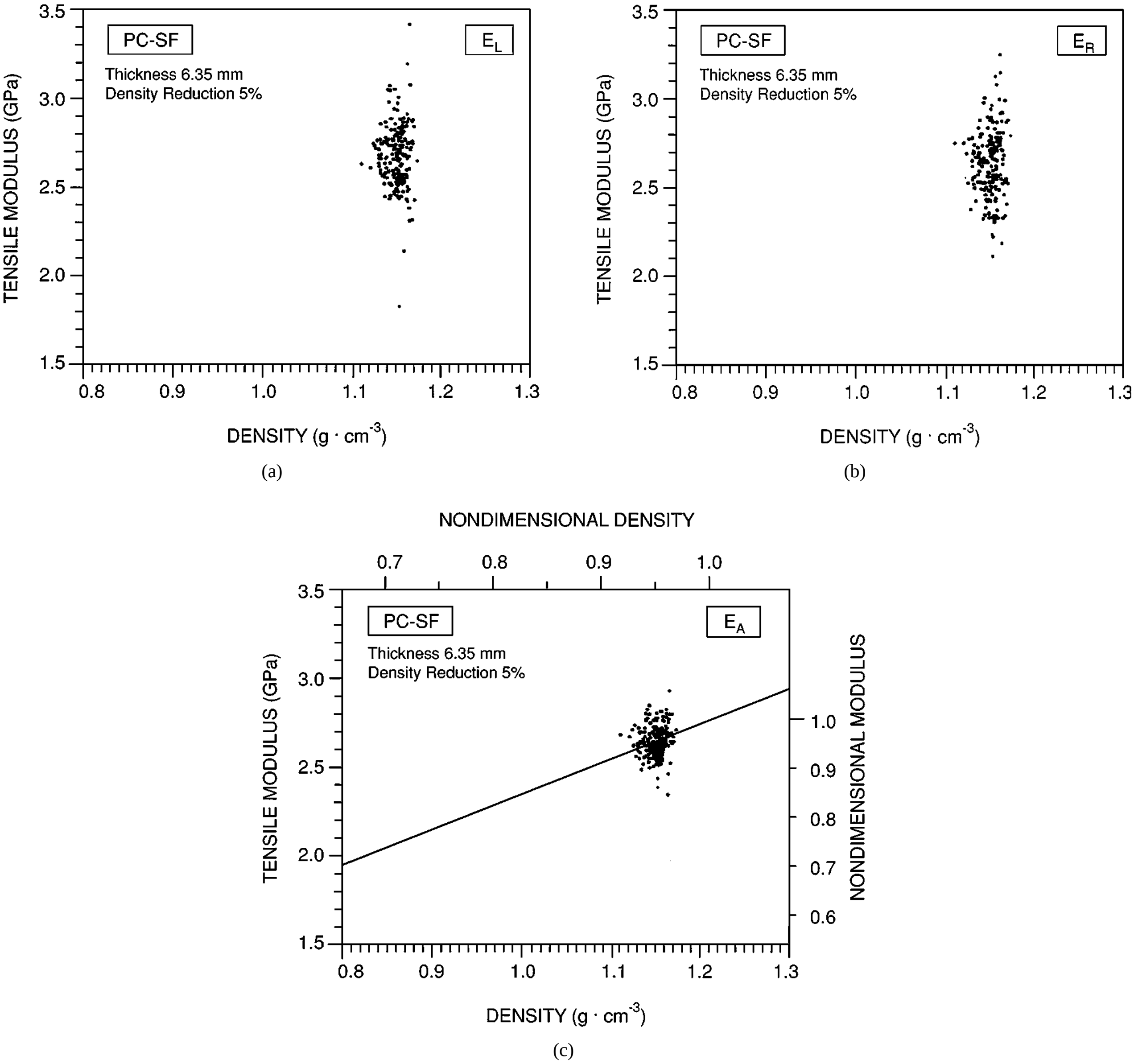


Figure 6 Variations of the Young's moduli versus the density ρ over a 101.6 × 304.8-mm area of a 6.35-mm-thick, 5%-density-reduction foam plaque. (a) Left modulus E_L, (b) right modulus E_R, and (c) average modulus E_A.

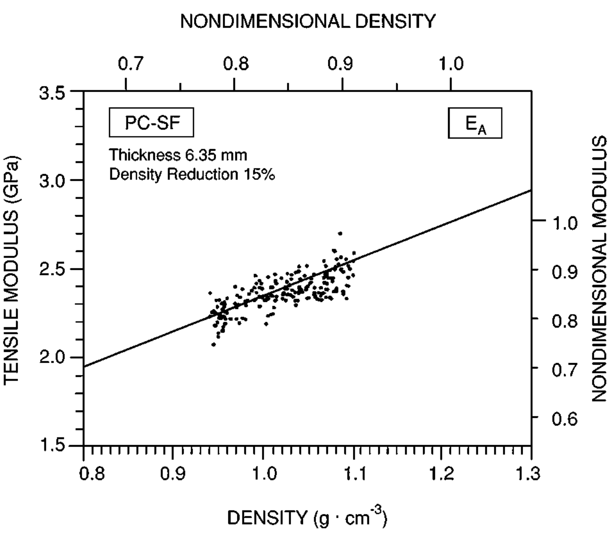


Figure 7 Variation of the average Young's modulus E_A versus the density ρ over a 101.6 × 304.8-mm area of a 15%-density-reduction foam plaque.

(specimens 1 through 3 and 7, segments 1; specimen 2, segment 2). The ratio of the maximum to the minimum density is 1.18. This range of variation, 0.88–1.04 g · cm^{-3}, overlaps the density variation range, 0.94–1.10 g · cm^{-3}, in the 15%-density reduction plaque.

The variations of E_L, E_R, E_A, and ρ along specimens 1, 4, 5, and 8 are shown in Fig. 8. Clearly, the differences between E_L and E_R at a segment can be much larger than for the 5%-density-reduction material (Fig. 5). Furthermore, E_A exhibits a decreasing trend with distance from the gated end. Here again, E_A appears to track the variation in the density ρ.

Fig. 9a–c are plots of the local left, right, and average moduli E_L, E_R, and E_A, respectively, versus the local density ρ (data from Tables VI and VII): First, the densities are distributed over a wider band of 0.88 to 1.04 g · cm^{-3}. The spread of 0.16 g · cm^{-3} is much larger than the spread of 0.06 g · cm^{-3} in the 5%-density-reduction foam (Fig. 5). Fig. 9a clearly shows

TABLE IV Variation of the left and right flow-direction tensile moduli over a 6.35-mm-thick, 15%-density-reduction polycarbonate structural foam plaque

Segment number	Tensile modulus (GPa)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R
1	2.99	2.13	3.01	2.18	2.33	2.77	2.59	2.63	**3.20**	1.90	2.37	2.61	2.32	2.62	2.04	**2.87**
2	3.03	2.37	2.89	2.16	2.46	2.55	2.18	2.58	3.12	1.92	2.23	2.76	2.36	2.45	2.17	2.76
3	2.90	2.30	2.63	2.12	2.43	2.39	2.33	2.46	2.98	2.14	2.25	2.76	2.33	2.54	2.17	2.83
4	2.84	2.22	2.55	2.17	2.35	2.33	2.38	2.29	2.87	2.10	2.36	2.65	2.32	2.45	2.35	2.68
5	2.78	2.13	2.48	2.37	2.41	2.34	2.36	2.33	2.76	2.08	2.32	2.59	2.35	2.64	2.42	2.72
6	2.78	2.17	2.43	2.23	2.39	2.34	2.25	2.41	2.66	2.06	2.34	2.56	2.32	2.59	2.15	2.52
7	2.69	2.21	2.44	2.29	2.54	2.23	2.23	2.47	2.58	2.22	2.21	2.48	2.35	2.57	2.21	2.58
8	2.72	2.13	2.52	2.22	2.53	2.29	2.43	2.26	2.53	2.36	2.39	2.54	2.35	2.58	2.17	2.50
9	2.65	2.19	2.46	2.28	2.40	2.29	2.45	2.20	2.49	2.29	2.26	2.54	2.39	2.56	2.22	2.58
10	2.83	2.22	2.41	2.39	2.39	2.28	2.46	2.29	2.43	2.54	2.32	2.43	2.26	2.48	2.22	2.53
11	2.68	2.30	2.38	2.22	2.34	2.43	2.29	2.36	2.42	2.31	2.37	2.46	2.35	2.68	2.27	2.53
12	2.48	2.28	2.41	2.30	2.37	2.43	2.38	2.33	2.37	2.41	2.37	2.33	2.28	2.70	2.22	2.43
13	2.35	2.37	2.38	2.51	2.37	2.57	2.34	2.33	2.37	2.33	2.41	2.37	2.32	2.54	2.28	2.42
14	2.25	2.40	2.38	2.34	2.41	2.54	2.40	2.40	2.41	2.46	2.54	2.41	2.34	2.56	2.18	2.40
15	2.25	2.43	2.32	2.18	2.41	2.43	2.46	2.25	2.38	2.38	2.41	2.30	2.34	2.58	2.24	2.42
16	2.39	2.41	2.25	2.23	2.33	2.51	2.59	2.06	2.32	2.34	2.41	2.28	2.22	2.58	2.08	2.30
17	2.19	2.54	2.30	2.28	2.37	2.30	2.52	1.98	2.40	2.27	2.43	2.22	2.30	2.50	2.15	2.28
18	2.24	2.41	2.37	2.33	2.53	2.31	2.60	2.09	2.51	2.27	2.30	2.28	2.32	2.56	2.28	2.44
19	2.32	2.49	2.19	2.26	2.51	2.42	2.71	1.89	2.41	2.19	2.19	2.25	2.28	2.35	2.18	2.22
20	2.29	2.36	2.28	2.30	2.58	2.20	2.92	1.79	2.43	2.01	2.12	2.32	2.20	2.42	2.02	2.28
21	2.42	2.31	2.27	2.12	2.56	2.12	2.70	1.82	2.49	2.12	2.21	2.30	2.27	2.29	2.09	2.38
22	2.18	2.29	2.18	2.14	2.53	2.05	2.81	1.82	2.52	1.99	2.08	2.32	2.31	2.27	**2.00**	2.40
23	2.22	2.30	2.29	2.08	2.54	1.92	2.91	**1.64**	2.41	1.95	2.04	2.41	2.22	2.26	1.81	2.43
24	2.15	2.31	2.22	2.07	2.51	2.02	2.89	1.73	2.52	1.93	1.97	2.50	2.33	2.33	1.72	2.43

$(E_L)_{\min} = 2.00$ GPa, $(E_L)_{\max} = 3.20$ GPa, $(E_R)_{\min} = 1.64$ GPa, $(E_R)_{\max} = 2.87$ GPa.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance:14.22 mm. Nominal segment centerline distance:12.7 mm.

TABLE V Variation of the average flow-direction tensile modulus E_A and the local density ρ over a 6.35-mm-thick, 15%-density-reduction polycarbonate structural foam plaque

Segment number	Average tensile modulus (GPa) and local density ($g \cdot cm^{-3}$)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ
1	2.56	1.09	2.59	1.10	2.55	1.10	2.61	1.08	2.55	1.07	2.49	1.09	2.47	1.10	2.46	1.09
2	**2.70**	1.09	2.52	1.09	2.50	1.10	2.38	1.08	2.52	1.07	2.50	1.09	2.41	1.09	2.47	1.07
3	2.60	1.08	2.38	1.09	2.41	1.09	2.39	1.08	2.56	1.08	2.50	1.09	2.43	1.08	2.50	1.08
4	2.53	1.08	2.36	1.09	2.34	1.09	2.34	1.06	2.48	1.07	2.50	1.08	2.39	1.07	2.52	1.08
5	2.45	1.07	2.43	1.07	2.38	1.08	2.34	1.07	2.42	1.07	2.45	1.07	2.50	1.07	2.57	1.09
6	2.47	1.07	2.33	1.06	2.37	1.07	2.33	1.07	2.36	1.06	2.45	1.08	2.45	1.07	2.33	1.06
7	2.45	1.05	2.37	1.07	2.39	1.07	2.35	1.06	2.40	1.06	2.34	1.07	2.46	1.07	2.40	1.06
8	2.43	1.05	2.37	1.06	2.41	1.05	2.34	1.05	2.45	1.04	2.47	1.05	2.46	1.07	2.34	1.08
9	2.42	1.05	2.37	1.06	2.35	1.06	2.32	1.04	2.39	1.05	2.40	1.04	2.47	1.06	2.40	1.05
10	2.52	1.05	2.40	1.04	2.34	1.05	2.37	1.04	2.48	1.04	2.37	1.04	2.37	1.04	2.38	1.04
11	2.49	1.04	2.30	1.04	2.39	1.04	2.32	1.04	2.37	1.02	2.41	1.03	2.52	1.04	2.40	1.04
12	2.38	1.03	2.36	1.03	2.40	1.02	2.36	1.03	2.39	1.03	2.35	1.03	2.49	1.04	2.32	1.03
13	2.36	1.03	2.45	1.03	2.47	1.03	2.34	1.02	2.35	1.01	2.39	1.02	2.43	1.03	2.35	1.00
14	2.33	1.01	2.36	1.02	2.48	1.03	2.40	1.01	2.43	1.01	2.47	1.01	2.45	1.02	2.29	1.01
15	2.34	1.00	2.25	1.01	2.42	1.01	2.35	1.01	2.38	0.99	2.35	1.01	2.46	1.01	2.33	1.00
16	2.40	1.00	2.24	1.01	2.42	1.00	2.33	1.00	2.33	0.99	2.35	1.00	2.40	0.99	2.19	1.00
17	2.37	0.99	2.29	1.00	2.33	0.99	2.25	1.01	2.34	1.01	2.32	1.01	2.40	1.01	2.22	0.98
18	2.33	0.98	2.35	1.00	2.42	0.99	2.34	0.97	2.39	1.00	2.29	1.00	2.44	1.00	2.36	0.98
19	2.41	0.97	2.23	0.98	2.47	0.99	2.30	0.97	2.30	0.98	2.22	0.97	2.32	0.98	2.20	0.97
20	2.33	0.95	2.29	0.97	2.39	0.98	2.35	0.97	2.22	0.96	2.22	0.96	2.31	0.97	2.15	0.95
21	2.37	0.94	2.20	0.96	2.34	0.96	2.26	0.96	2.30	0.96	2.25	0.96	2.28	0.95	2.23	0.96
22	2.23	0.94	2.16	0.95	2.29	0.95	2.32	0.96	2.25	0.95	2.20	0.96	2.29	0.96	2.20	0.95
23	2.26	0.94	2.18	0.95	2.23	0.95	2.27	0.95	2.18	0.96	2.23	0.95	2.24	0.95	2.12	0.95
24	2.23	0.95	2.14	0.95	2.26	0.94	2.31	0.95	2.23	0.95	2.24	0.95	2.33	0.95	**2.08**	0.94

$(E_A)_{\min} = 2.08$ GPa, $(E_A)_{\max} = 2.70$ GPa, $(\rho)_{\min} = 0.94\ g \cdot cm^{-3}$, $(\rho)_{\max} = 1.10\ g \cdot cm^{-3}$.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

TABLE VI Variation of the left and right flow-direction tensile moduli over a 6.35-mm-thick, 25%-density-reduction polycarbonate structural foam plaque

Segment number	Tensile modulus (GPa)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R
1	2.19	2.43	2.32	1.97	**3.84**	1.59	2.84	2.05	3.77	**1.55**	2.48	2.39	2.34	2.56	2.97	1.93
2	2.19	2.41	2.24	2.09	3.39	1.62	2.69	2.00	3.28	1.62	2.43	2.22	2.30	2.46	2.81	1.93
3	2.13	2.37	2.20	2.42	3.22	1.71	2.67	2.14	2.96	1.83	2.55	2.35	2.33	2.49	2.69	2.11
4	2.22	2.33	2.20	2.42	2.86	1.82	2.43	2.16	2.76	1.99	2.40	2.31	2.22	2.44	2.75	1.93
5	2.34	2.29	2.31	2.47	2.80	1.83	2.56	2.21	2.61	1.86	2.40	2.40	2.22	2.46	2.75	2.11
6	2.35	2.33	2.24	2.46	2.75	1.96	2.50	2.12	2.61	1.95	2.34	2.36	2.15	2.51	2.75	2.02
7	2.34	2.30	2.25	2.51	2.57	2.08	2.48	2.26	2.55	2.09	2.41	2.38	2.19	2.45	2.62	2.13
8	2.46	2.37	2.25	2.53	2.47	2.14	2.48	2.30	2.57	2.02	2.31	2.38	2.10	2.52	2.64	2.04
9	2.45	2.32	2.36	2.53	2.30	2.25	2.39	2.30	2.54	2.06	2.29	2.29	2.14	2.45	2.46	2.17
10	2.36	2.32	2.20	2.44	2.24	2.22	2.41	2.23	2.55	2.08	2.26	2.35	2.19	2.24	2.33	2.31
11	2.27	2.34	2.35	2.35	2.10	2.32	2.42	2.42	2.41	2.14	2.26	2.44	2.22	2.30	2.26	2.41
12	2.38	2.33	2.40	2.31	2.21	2.34	2.42	2.31	2.36	2.16	2.23	2.45	2.21	2.23	2.19	2.35
13	2.24	2.28	2.53	2.33	2.15	2.44	2.42	2.22	2.44	2.07	2.18	2.33	2.22	2.20	2.43	**2.75**
14	2.24	2.33	2.38	2.29	2.18	2.43	2.51	2.20	2.42	2.11	2.29	2.22	2.24	2.28	2.19	2.43
15	2.29	2.13	2.57	2.17	2.11	2.26	2.42	2.11	2.42	2.09	2.25	2.27	2.19	2.22	2.13	2.39
16	2.31	2.14	2.48	2.22	2.20	2.18	2.53	1.98	2.28	2.15	2.22	2.25	2.13	2.20	2.13	2.33
17	2.29	2.20	2.49	2.01	2.41	2.08	2.57	1.96	2.22	2.18	2.26	2.12	2.15	2.11	2.06	2.41
18	2.24	2.15	2.54	2.07	2.36	2.14	2.46	2.06	2.09	2.27	2.43	2.06	2.26	2.17	2.02	2.33
19	2.34	2.10	2.60	1.96	2.16	2.01	2.37	2.13	2.09	2.24	2.32	2.01	2.24	2.08	2.05	2.36
20	2.33	2.00	2.60	1.94	2.24	2.00	2.17	2.19	2.20	2.31	2.41	1.88	2.21	1.99	2.09	2.24
21	2.35	2.00	2.69	1.83	2.13	1.96	2.18	2.11	2.41	2.25	2.47	1.81	2.23	1.99	2.10	2.24
22	2.52	2.01	2.70	1.86	2.23	2.06	2.33	2.09	**1.94**	2.30	2.56	1.70	2.36	1.92	2.12	2.23
23	2.50	1.90	2.62	1.86	2.17	2.04	2.43	2.08	2.30	2.08	2.76	1.68	2.34	1.99	1.99	2.21
24	2.55	2.06	2.56	1.96	2.35	1.89	2.38	2.07	2.73	2.00	2.95	**1.55**	2.48	1.90	1.96	2.47

$(E_L)_{min} = 1.94$ GPa, $(E_L)_{max} = 3.84$ GPa, $(E_R)_{min} = 1.55$ GPa, $(E_R)_{max} = 2.75$ GPa.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

TABLE VII Variation of the average flow-direction tensile modulus E_A and the local density ρ over a 6.35-mm-thick, 25%-density-reduction polycarbonate structural foam plaque

Segment number	Average tensile modulus (GPa) and local density ($g \cdot cm^{-3}$)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ
1	2.31	1.04	2.15	1.04	**2.71**	1.04	2.44	1.02	2.66	1.02	2.43	1.03	2.45	1.04	2.45	1.02
2	2.30	1.03	2.16	1.04	2.50	1.03	2.34	1.02	2.45	1.01	2.32	1.03	2.38	1.03	2.37	1.03
3	2.25	1.02	2.31	1.02	2.47	1.03	2.41	1.02	2.39	1.00	2.45	1.03	2.41	1.02	2.40	1.02
4	2.27	1.02	2.31	1.02	2.34	1.02	2.29	1.00	2.37	1.00	2.36	1.01	2.33	1.01	2.34	1.00
5	2.32	1.02	2.39	1.02	2.31	1.01	2.38	1.00	2.23	0.99	2.40	1.00	2.34	1.01	2.43	1.01
6	2.34	1.01	2.35	1.01	2.35	1.01	2.31	0.99	2.28	0.99	2.35	1.00	2.33	1.00	2.39	1.00
7	2.32	1.00	2.38	1.00	2.33	1.00	2.37	1.00	2.32	0.98	2.39	0.98	2.32	1.00	2.38	1.00
8	2.42	1.00	2.39	0.99	2.30	0.98	2.39	0.99	2.30	0.97	2.34	0.99	2.31	0.99	2.34	0.99
9	2.39	0.98	2.44	0.98	2.28	0.97	2.34	0.98	2.30	0.97	2.29	0.97	2.30	0.99	2.32	0.99
10	2.34	0.99	2.32	0.99	2.23	0.98	2.32	0.97	2.31	0.96	2.30	0.97	2.22	0.98	2.32	0.98
11	2.30	0.97	2.35	0.98	2.21	0.97	2.42	0.97	2.27	0.96	2.35	0.97	2.26	0.96	2.34	0.98
12	2.35	0.97	2.35	0.97	2.28	0.97	2.36	0.97	2.26	0.96	2.34	0.97	2.22	0.97	2.27	0.97
13	2.26	0.96	2.43	0.96	2.29	0.96	2.32	0.96	2.26	0.96	2.26	0.96	2.21	0.96	2.59	0.96
14	2.28	0.96	2.33	0.96	2.31	0.96	2.36	0.96	2.26	0.96	2.26	0.95	2.26	0.96	2.31	0.96
15	2.21	0.95	2.37	0.96	2.19	0.94	2.27	0.95	2.26	0.95	2.26	0.94	2.20	0.95	2.26	0.95
16	2.22	0.95	2.35	0.95	2.19	0.94	2.25	0.94	2.22	0.93	2.23	0.94	2.16	0.93	2.23	0.94
17	2.25	0.94	2.25	0.93	2.24	0.94	2.27	0.93	2.20	0.91	2.19	0.93	2.13	0.94	2.23	0.94
18	2.20	0.93	2.31	0.93	2.25	0.94	2.26	0.93	2.18	0.93	2.24	0.93	2.22	0.93	2.18	0.93
19	2.22	0.91	2.28	0.91	2.09	0.91	2.25	0.92	2.16	0.90	2.17	0.92	2.16	0.90	2.20	0.90
20	2.16	0.91	2.27	0.91	2.12	0.90	2.18	0.90	2.26	0.94	2.14	0.90	2.10	0.89	2.16	0.91
21	2.18	0.91	2.26	0.89	**2.04**	0.90	2.15	0.90	2.33	0.90	2.14	0.90	2.11	0.88	2.17	0.91
22	2.26	0.90	2.28	0.91	2.15	0.90	2.21	0.90	2.12	0.88	2.13	0.90	2.14	0.89	2.17	0.90
23	2.20	0.90	2.24	0.89	2.10	0.91	2.26	0.93	2.19	0.90	2.22	0.91	2.16	0.89	2.10	0.92
24	2.30	0.93	2.26	0.90	2.12	0.92	2.23	0.93	2.36	0.93	2.25	0.89	2.19	0.92	2.21	0.91

$(E_A)_{min} = 2.04$ GPa, $(E_A)_{max} = 2.71$ GPa, $(\rho)_{min} = 0.88\ g \cdot cm^{-3}$, $(\rho)_{max} = 1.04\ g \cdot cm^{-3}$.
Nominal specimen thickness: 6.35 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

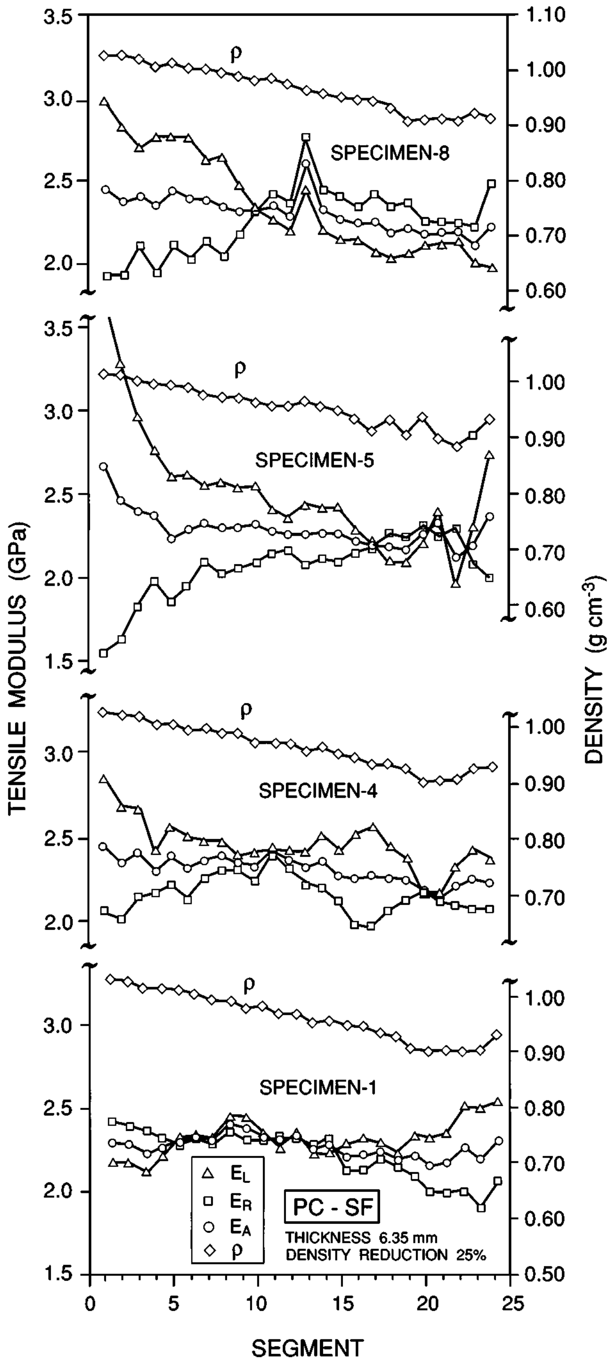


Figure 8 Variations of the Young's moduli E_L, E_R, and E_A and the density ρ at 12.7-mm intervals along specimens 1, 4, 5, and 8 from a 6.35-mm-thick, 25%-density-reduction foam plaque.

that E_L does not correlate with ρ. (Note that the two data points for $E_L = 3.84$ and 3.77 GPa, corresponding to segment 1 in specimens 3 and 5, respectively, are not included in this figure.) Second, Fig. 9c shows that E_A correlates strongly with ρ, the dependence being approximately linear.

4.4. Density-modulus correlation for 6.35-mm-thick foam

The consolidated data on variations of the average tensile modulus E_A versus the local density ρ for all three nominal density reductions of 5, 15, and 25%—from Figs 6c, 7, and 9c, respectively—are shown in Fig. 10. This figure shows that, in 6.35-mm-thick plaques with nominal density reductions of 15 and 25%, regions with the same local density have the same tensile modulus within the inherent scatter in this class of materials. Also, the data for the 5% nominal density material follows the same linear dependence of the modulus on the density, even though the local density variations in this material do not overlap those in the 15 and 25% density reduction cases. Thus, the *local* average modulus correlates with the *local* density, independent of whether the local properties are for a 5, 15, or 25%-density-reduction foam. This empirical result is important from the perspective of mechanical design, as it suggests that the local material stiffness can be determined once the local density is known. Of course, this study has not considered any anisotropy introduced by the cell structure being affected by the flow field—different cell shapes in the flow and cross flow directions. Note that the straight line drawn through the data in this figure, $E_A = 2\rho + 0.35$, was not obtained through a least squares fit; rather, it represents a visual fit.

4.5. Tensile modulus and density data for 4-mm-thick, 5% density reduction foam

The local tensile moduli E_L and E_R for one 4-mm-thick, 5%-density-reduction plaque, are listed in Table VIII. The left modulus E_L varies from a minimum of 2.14 GPa (specimen 7, segments 23 and 24) to a maximum of 3.25 GPa (specimen 2, segment 1), resulting in a ratio of maximum to minimum of 1.52. The corresponding numbers for the right modulus E_R are 1.99 GPa (specimen 4, segment 24), 3.58 GPa (specimen 1, segment 1), and 1.80, respectively. And, from Table IX, those for E_A are 2.23 GPa (specimen 3, segment 24), 3.10 GPa (specimen 8, segment 1) and 1.39, respectively.

Local densities are also listed in Table IX. The density varies from a minimum of 0.97 $g \cdot cm^{-3}$ (specimen 5, segment 24) to a maximum of 1.18 $g \cdot cm^{-3}$ (specimen 6, segment 1, and specimen 7, segment10), the ratio of the maximum to the minimum being 1.22. A comparison with the data in Table III shows that the 4-mm-thick material has a larger variation than that (1.11–1.17 $g \cdot cm^{-3}$) in the 6.35-mm-thick material.

A comparison of Fig. 6a with the variation of local left tensile modulus E_L (data from Tables VIII and IX) in Fig. 11a shows that the 4-mm-thick material also has large variations in E_L; it also exhibits a larger local density variation. Fig. 11b shows the variation of E_A versus ρ. Although the E_A again appears to correlate linearly with ρ, the data are above the solid line $E_A = 2\rho + 0.35$ used to correlate the data for the 6.35-mm-thick material.

4.6. Tensile modulus and density data for 4-mm-thick, 15% density reduction foam

The local tensile moduli E_L and E_R for one 15%-density-reduction plaque are listed in Table X. The left

TABLE VIII Variation of the left and right flow-direction tensile moduli over a 4-mm-thick, 5%-density-reduction polycarbonate structural foam plaque

Segment number	Tensile modulus (GPa) — Specimen number 1		2		3		4		5		6		7		8	
	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R
1	2.36	**3.58**	**3.25**	2.69	2.32	3.40	2.78	3.09	2.46	3.01	2.46	3.45	2.61	3.06	2.63	3.39
2	2.30	3.41	3.09	2.63	2.55	3.21	2.54	3.03	2.49	2.98	2.38	3.10	2.89	2.89	2.61	3.33
3	2.31	3.29	2.93	2.61	2.62	3.11	2.59	2.94	2.50	2.85	2.37	2.89	2.97	2.79	2.62	3.35
4	2.36	3.17	2.89	2.58	2.62	2.97	2.57	2.92	2.55	2.83	2.45	2.80	2.99	2.57	2.59	3.05
5	2.45	2.98	2.68	2.61	2.63	2.95	2.58	2.79	2.50	2.71	2.54	2.75	3.03	2.71	2.51	3.04
6	2.36	2.93	2.68	2.54	2.55	2.91	2.67	2.74	2.58	2.73	2.50	2.71	3.02	2.77	2.54	3.04
7	2.33	3.01	2.62	2.55	2.73	2.84	2.71	2.71	2.53	2.82	2.57	2.68	3.00	2.72	2.61	3.01
8	2.25	3.15	2.67	2.64	2.61	2.79	2.69	2.73	2.53	2.67	2.52	2.62	3.00	2.74	2.66	3.03
9	2.41	2.77	2.71	2.64	2.68	2.79	2.72	2.83	2.62	2.76	2.63	2.63	2.96	2.63	2.63	2.88
10	2.60	2.96	2.75	2.68	2.80	2.91	2.72	2.76	2.62	2.76	2.74	2.70	2.96	2.74	2.61	3.01
11	2.56	3.03	2.82	2.71	2.79	2.97	2.83	2.76	2.72	2.76	2.81	2.74	2.98	2.76	2.65	2.94
12	2.60	2.96	2.85	2.74	2.75	2.97	2.90	2.75	2.79	2.72	2.89	2.74	2.83	2.87	2.72	2.93
13	2.46	2.89	2.85	2.71	2.66	2.88	2.96	2.64	2.69	2.76	2.91	2.80	2.83	2.79	2.65	2.90
14	2.64	2.85	2.89	2.67	2.73	2.84	2.96	2.64	2.71	2.74	2.94	2.72	2.87	2.83	2.72	2.79
15	2.73	2.84	2.78	2.71	2.83	2.79	2.93	2.53	2.74	2.64	2.80	2.72	2.75	2.90	2.83	2.72
16	2.73	2.76	2.74	2.67	2.72	2.90	2.94	2.43	2.60	2.74	2.83	2.65	2.66	2.88	2.92	2.56
17	2.73	2.69	2.87	2.61	2.70	2.80	2.93	2.62	2.85	2.74	2.85	2.78	2.58	2.88	2.91	2.55
18	2.84	2.88	2.87	2.76	2.77	2.77	2.92	2.46	2.74	2.67	2.83	2.83	2.57	2.90	3.09	2.40
19	2.77	2.80	2.79	2.65	2.66	2.77	2.96	2.60	2.70	2.74	2.83	2.80	2.47	2.99	3.02	2.33
20	2.80	2.51	2.94	2.58	2.70	2.70	2.87	2.40	2.78	2.67	2.82	2.78	2.51	3.03	3.08	2.23
21	2.75	2.42	2.78	2.59	2.68	2.71	2.87	2.33	2.62	2.58	2.64	2.75	2.37	3.00	3.08	2.16
22	2.79	2.16	2.66	2.44	2.46	2.46	2.64	2.09	2.73	2.36	2.62	2.58	2.23	2.83	2.95	2.03
23	2.86	2.29	2.79	2.42	2.40	2.33	2.60	2.01	2.62	2.25	2.49	2.57	**2.14**	2.70	3.06	2.03
24	2.98	2.10	2.46	2.31	2.28	2.17	2.72	**1.99**	2.61	2.09	2.25	2.43	**2.14**	2.82	2.68	2.05

$(E_L)_{\min} = 2.14$ GPa, $(E_L)_{\max} = 3.25$ GPa, $(E_R)_{\min} = 1.99$ GPa, $(E_R)_{\max} = 3.58$ GPa.
Nominal specimen thickness: 4 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

TABLE IX Variation of the average flow-direction tensile modulus E_A and the local density ρ over a 4-mm-thick, 5%-density-reduction polycarbonate structural foam plaque

Segment number	Average tensile modulus (GPa) and local density ($g \cdot cm^{-3}$) — Specimen number 1		2		3		4		5		6		7		8	
	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ
1	2.97	1.17	2.97	1.17	2.86	1.13	2.94	1.15	2.73	1.14	2.96	1.18	2.84	1.17	**3.01**	1.14
2	2.86	1.16	2.86	1.16	2.88	1.14	2.78	1.15	2.73	1.14	2.74	1.17	2.89	1.17	2.97	1.15
3	2.80	1.16	2.77	1.16	2.86	1.15	2.76	1.14	2.67	1.14	2.63	1.17	2.88	1.17	2.98	1.14
4	2.77	1.15	2.73	1.15	2.79	1.14	2.74	1.14	2.69	1.14	2.63	1.16	2.78	1.16	2.82	1.14
5	2.72	1.14	2.64	1.15	2.79	1.13	2.69	1.14	2.60	1.14	2.64	1.15	2.87	1.17	2.77	1.11
6	2.64	1.15	2.61	1.14	2.73	1.14	2.71	1.13	2.66	1.14	2.61	1.14	2.90	1.17	2.79	1.14
7	2.67	1.15	2.58	1.15	2.79	1.13	2.71	1.15	2.67	1.15	2.62	1.16	2.86	1.17	2.81	1.14
8	2.70	1.15	2.66	1.15	2.70	1.13	2.71	1.15	2.60	1.14	2.57	1.15	2.87	1.17	2.85	1.14
9	2.59	1.14	2.67	1.15	2.74	1.16	2.78	1.14	2.69	1.15	2.63	1.15	2.80	1.17	2.76	1.13
10	2.78	1.13	2.71	1.15	2.85	1.15	2.74	1.14	2.69	1.14	2.72	1.15	2.85	1.18	2.81	1.13
11	2.79	1.14	2.76	1.15	2.88	1.16	2.79	1.15	2.74	1.14	2.77	1.15	2.87	1.16	2.80	1.14
12	2.78	1.13	2.80	1.14	2.86	1.15	2.82	1.14	2.76	1.13	2.81	1.16	2.85	1.15	2.83	1.13
13	2.67	1.14	2.78	1.15	2.77	1.15	2.80	1.13	2.73	1.14	2.85	1.16	2.81	1.15	2.77	1.13
14	2.75	1.13	2.78	1.13	2.78	1.15	2.80	1.12	2.73	1.13	2.83	1.16	2.85	1.15	2.75	1.13
15	2.79	1.11	2.74	1.13	2.81	1.15	2.73	1.12	2.69	1.13	2.76	1.15	2.82	1.14	2.78	1.12
16	2.74	1.13	2.70	1.11	2.81	1.13	2.68	1.11	2.67	1.13	2.74	1.15	2.77	1.15	2.74	1.13
17	2.71	1.13	2.74	1.12	2.75	1.15	2.78	1.10	2.80	1.13	2.81	1.15	2.73	1.14	2.73	1.12
18	2.86	1.12	2.81	1.13	2.77	1.10	2.69	1.08	2.70	1.13	2.83	1.14	2.73	1.15	2.75	1.12
19	2.79	1.12	2.72	1.11	2.71	1.12	2.78	1.10	2.72	1.12	2.81	1.13	2.73	1.13	2.67	1.11
20	2.65	1.10	2.76	1.10	2.70	1.10	2.64	1.09	2.73	1.06	2.80	1.13	2.77	1.12	2.66	1.10
21	2.59	1.09	2.68	1.09	2.70	1.09	2.60	1.07	2.60	1.05	2.69	1.13	2.68	1.12	2.62	1.10
22	2.48	1.06	2.55	1.06	2.46	1.06	2.37	1.05	2.55	1.01	2.60	1.10	2.53	1.09	2.49	1.07
23	2.57	1.01	2.61	1.03	2.36	1.02	2.31	1.02	2.44	0.99	2.53	1.06	2.42	1.06	2.54	1.04
24	2.54	1.02	2.39	1.02	**2.23**	1.01	2.35	1.01	2.35	0.97	2.34	1.04	2.48	1.03	2.36	1.02

$(E_A)_{\min} = 2.23$ GPa, $(E_A)_{\max} = 3.10$ GPa, $(\rho)_{\min} = 0.97\ g \cdot cm^{-3}$, $(\rho)_{\max} = 1.18\ g \cdot cm^{-3}$.
Nominal specimen thickness: 4 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

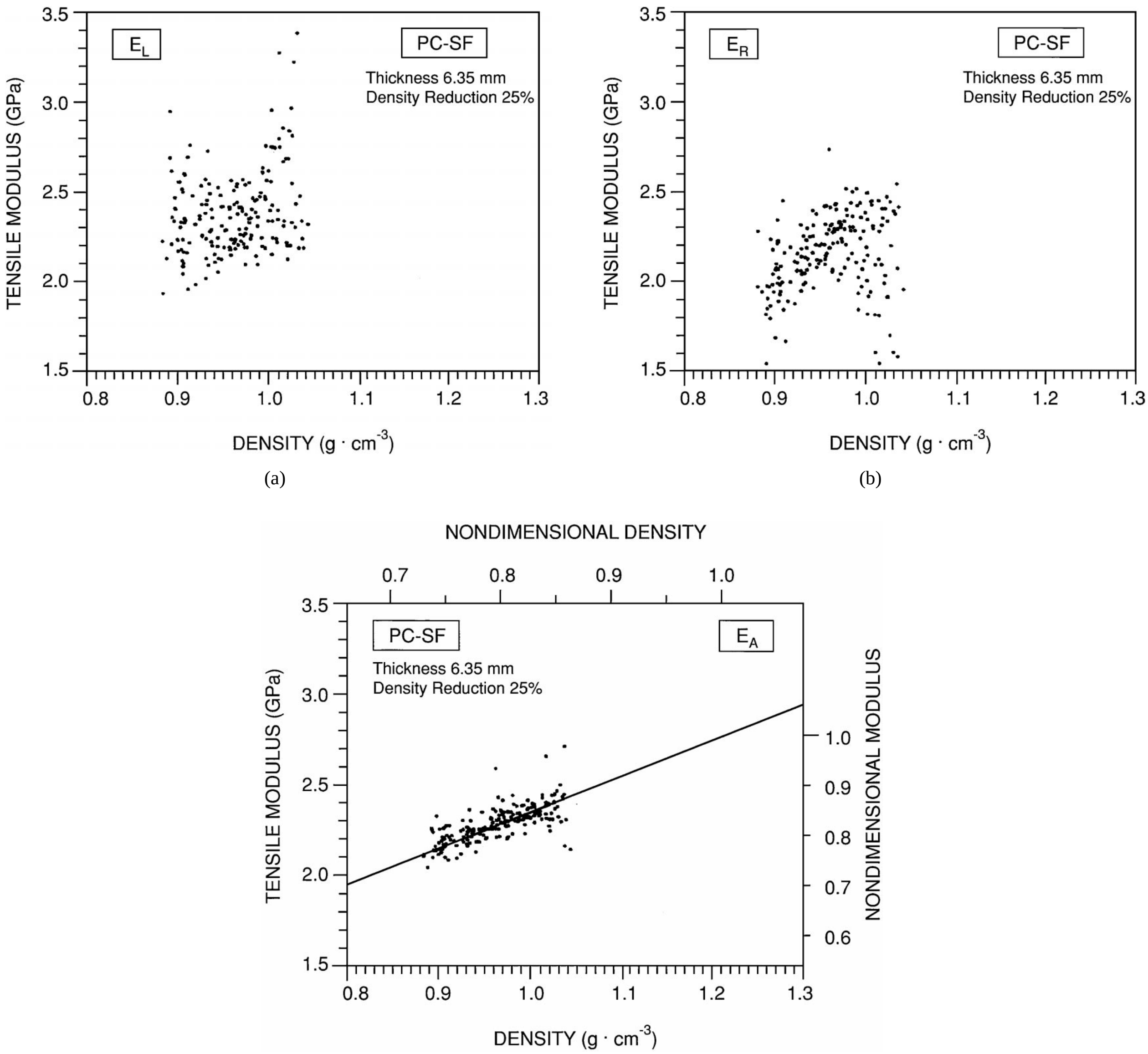


Figure 9 Variation of the Young's moduli versus the density ρ over a 101.6 × 304.8-mm area of a 6.35-mm-thick, 25%-density-reduction foam plaque. (a) Left modulus E_L, (b) right modulus E_R, and (c) average modulus E_A.

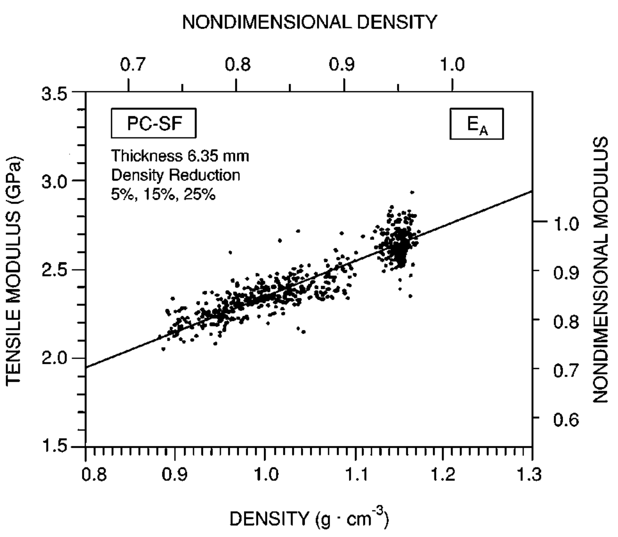


Figure 10 Variation of the average tensile modulus E_A versus the local density ρ over a 101.6 × 304.8-mm area of 6.35-mm-thick foam plaques, one for each nominal density reduction of 5, 15, and 25%. (Superposition of data from Figs 6c, 7, and 9c.)

modulus E_L varies from a minimum of $E_L = 1.84$ GPa (specimen 1, segment 24) to a maximum of 3.46 GPa (specimen 6, segment 7), resulting in a ratio of maximum to minimum of 1.88. The corresponding numbers for the right modulus E_R are 1.84 GPa (specimen 2, segment 24), 3.10 GPa (specimen 6, segment 1), and 1.68, respectively. And the corresponding numbers for E_A (data listed in Table XI) are 2.02 GPa (specimen 1, segment 24), 2.82 GPa (specimen 7, segment 4) and 1.40, respectively.

The local minimum and maximum densities for this plaque (data listed in Table XI) are 0.88 (specimen 2, segment 24) and 1.15 $g \cdot cm^{-3}$ (in eleven segments). This density variation overlaps that in the 5% density reduction material. Also, this is a much larger local density variation than that in the 6.35-mm-thick, 15%-density-reduction plaque (0.94 to 1.10 $g \cdot cm^{-3}$).

The variations of E_L and E_A with the local density are shown, respectively, in Fig. 12a and b (from data in Tables X and XI). Fig. 12b shows that the average tensile moduli for the 15%-density-reduction material

TABLE X Variation of the left and right flow-direction tensile moduli over a 4-mm-Thick, 15%-density-reduction polycarbonate structural foam plaque

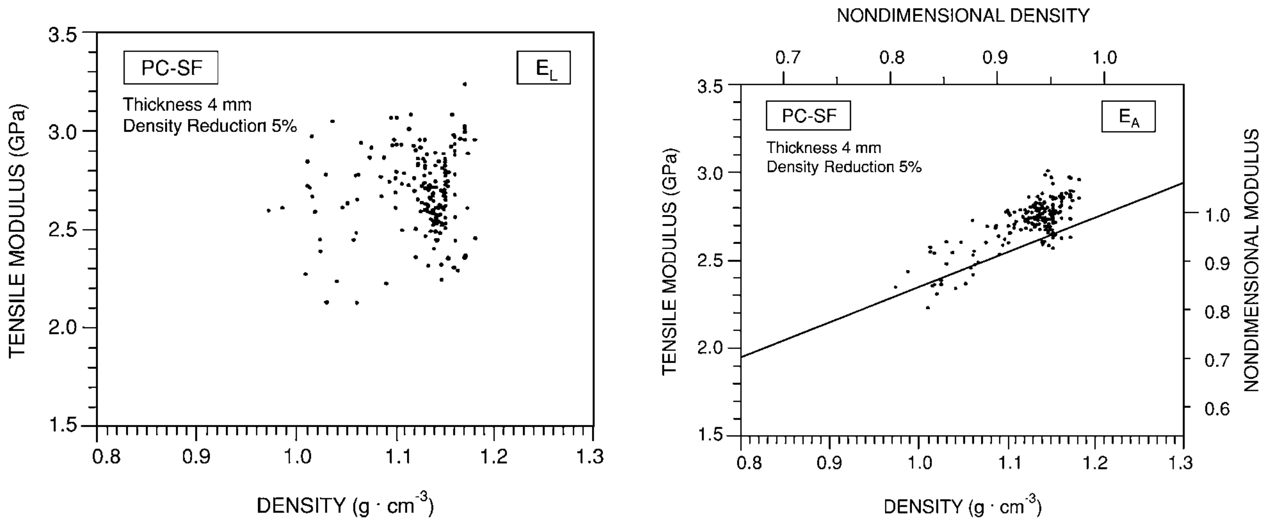
Segment number	Tensile modulus (GPa)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R	E_L	E_R
1	2.77	2.70	2.68	2.79	3.19	2.44	2.74	2.49	2.72	2.46	2.52	**3.10**	2.63	2.70	2.62	2.87
2	2.63	2.61	2.68	2.72	3.05	2.33	2.64	2.67	2.56	2.71	2.42	2.96	2.72	2.68	2.50	3.01
3	2.49	2.63	2.73	2.76	2.87	2.37	2.53	2.68	2.73	2.77	2.31	3.01	2.91	2.65	2.53	2.79
4	2.46	2.59	2.70	2.70	2.76	2.51	2.52	2.63	2.74	2.81	2.26	3.01	2.91	2.73	2.50	2.86
5	2.43	2.64	2.60	2.79	2.69	2.33	2.52	2.74	2.71	2.83	2.67	2.67	3.04	2.56	2.48	2.81
6	2.53	2.71	2.54	2.72	2.67	2.38	2.57	2.82	2.71	2.82	3.02	2.32	3.00	2.56	2.52	2.84
7	2.61	2.68	2.48	2.70	2.56	2.52	2.55	2.77	2.73	2.85	**3.46**	2.10	2.97	2.60	2.55	2.81
8	2.71	2.76	2.67	2.78	2.60	2.56	2.55	2.69	2.71	2.78	3.33	2.07	2.99	2.51	2.53	2.83
9	2.64	2.69	2.79	2.75	2.68	2.53	2.57	2.76	2.69	2.73	2.94	2.37	2.95	2.65	2.45	2.85
10	2.61	2.67	2.60	2.63	2.70	2.56	2.56	2.71	2.61	2.72	2.48	2.72	2.85	2.67	2.53	2.90
11	2.42	2.59	2.61	2.61	2.78	2.53	2.62	2.73	2.61	2.76	2.57	2.78	2.70	2.74	2.58	2.76
12	2.47	2.65	2.61	2.72	2.75	2.49	2.65	2.76	2.73	2.70	2.77	2.46	2.70	2.70	2.55	2.74
13	2.43	2.62	2.64	2.57	2.68	2.43	2.58	2.65	2.58	2.73	3.14	2.44	2.64	2.68	2.59	2.67
14	2.36	2.60	2.57	2.42	2.73	2.41	2.58	2.65	2.51	2.77	2.78	2.52	2.73	2.58	2.60	2.68
15	2.43	2.58	2.57	2.39	2.70	2.38	2.54	2.68	2.51	2.73	2.51	2.70	2.66	2.55	2.61	2.65
16	2.41	2.56	2.57	2.53	2.60	2.42	2.50	2.46	2.51	2.62	2.29	2.89	2.66	2.58	2.58	2.61
17	2.41	2.54	2.55	2.37	2.60	2.38	2.59	2.34	2.40	2.43	2.34	2.86	2.65	2.57	2.58	2.58
18	2.45	2.52	2.59	2.37	2.53	2.46	2.66	2.26	2.40	2.47	2.41	2.71	2.49	2.56	2.49	2.64
19	2.39	2.55	2.62	2.40	2.38	2.46	2.54	2.29	2.50	2.46	2.53	2.46	2.38	2.49	2.41	2.49
20	2.24	2.44	2.63	2.27	2.36	2.54	2.45	2.37	2.42	2.35	2.41	2.45	2.27	2.60	2.37	2.40
21	2.12	2.43	2.63	2.09	2.08	2.55	2.33	2.30	2.42	2.31	2.39	2.42	2.09	2.64	2.36	2.40
22	2.13	2.43	2.56	2.05	2.13	2.46	2.35	2.24	2.54	2.17	2.14	2.29	2.18	2.51	2.22	2.37
23	1.91	2.24	2.47	1.96	2.06	2.39	2.32	2.21	2.51	2.17	2.24	2.13	2.06	2.36	2.22	2.33
24	**1.84**	2.21	2.25	**1.84**	1.96	2.25	2.04	2.19	2.41	2.12	2.30	2.11	2.25	2.40	2.11	2.26

$(E_L)_{min} = 1.84$ GPa, $(E_L)_{max} = 3.46$ GPa, $(E_R)_{min} = 1.84$ GPa, $(E_R)_{max} = 3.10$ GPa.
Nominal specimen thickness: 4 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.

TABLE XI Variation of the average flow-direction tensile modulus E_A and the local density ρ over a 4-mm-thick, 15%-density-reduction polycarbonate structural foam plaque

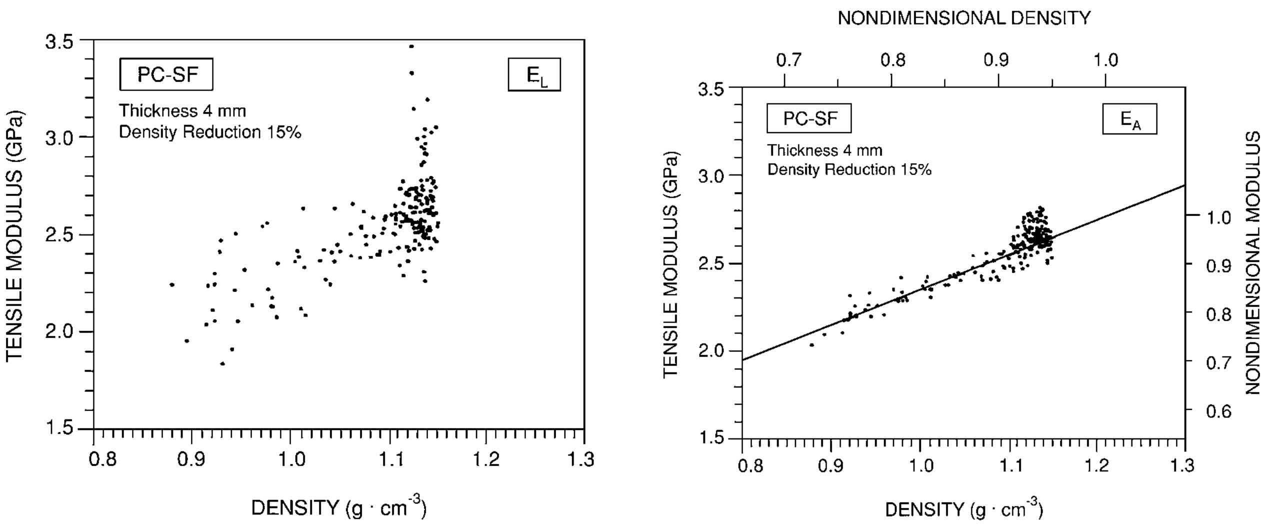
Segment number	Average tensile modulus (GPa) and local density ($g \cdot cm^{-3}$)															
	Specimen number															
	1		2		3		4		5		6		7		8	
	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ	E_A	ρ
1	2.74	1.15	2.74	1.14	2.81	1.14	2.61	1.15	2.59	1.13	2.81	1.14	2.66	1.14	2.75	1.14
2	2.62	1.14	2.70	1.14	2.69	1.15	2.66	1.14	2.64	1.13	2.69	1.13	2.70	1.14	2.75	1.14
3	2.56	1.13	2.74	1.13	2.62	1.14	2.61	1.14	2.75	1.12	2.66	1.14	2.78	1.14	2.66	1.13
4	2.53	1.15	2.70	1.14	2.64	1.14	2.57	1.13	2.78	1.13	2.64	1.14	**2.82**	1.14	2.68	1.14
5	2.53	1.14	2.69	1.14	2.51	1.15	2.63	1.14	2.77	1.12	2.67	1.13	2.80	1.14	2.64	1.14
6	2.62	1.13	2.63	1.15	2.53	1.15	2.70	1.14	2.76	1.12	2.67	1.14	2.78	1.14	2.68	1.13
7	2.65	1.14	2.59	1.15	2.54	1.15	2.66	1.13	2.79	1.12	2.78	1.12	2.78	1.14	2.68	1.13
8	2.73	1.13	2.72	1.14	2.58	1.15	2.62	1.13	2.74	1.12	2.70	1.12	2.75	1.13	2.68	1.13
9	2.67	1.12	2.77	1.14	2.60	1.14	2.66	1.13	2.71	1.13	2.66	1.14	2.80	1.13	2.65	1.13
10	2.64	1.13	2.62	1.15	2.63	1.13	2.64	1.13	2.67	1.11	2.60	1.14	2.76	1.13	2.71	1.13
11	2.51	1.13	2.61	1.14	2.65	1.14	2.67	1.14	2.68	1.12	2.68	1.12	2.72	1.12	2.67	1.12
12	2.56	1.13	2.66	1.15	2.62	1.13	2.71	1.13	2.71	1.11	2.62	1.11	2.70	1.12	2.64	1.12
13	2.53	1.12	2.60	1.13	2.55	1.13	2.61	1.12	2.66	1.11	2.79	1.13	2.66	1.12	2.63	1.12
14	2.48	1.12	2.50	1.12	2.57	1.11	2.61	1.12	2.64	1.13	2.65	1.13	2.66	1.12	2.64	1.11
15	2.50	1.11	2.48	1.12	2.54	1.12	2.61	1.11	2.62	1.10	2.60	1.13	2.60	1.14	2.63	1.13
16	2.49	1.12	2.55	1.11	2.51	1.12	2.48	1.11	2.56	1.09	2.59	1.11	2.62	1.12	2.60	1.11
17	2.47	1.10	2.46	1.11	2.49	1.10	2.46	1.08	2.41	1.09	2.60	1.11	2.61	1.11	2.58	1.10
18	2.48	1.08	2.48	1.10	2.49	1.09	2.46	1.06	2.43	1.09	2.56	1.09	2.53	1.08	2.57	1.08
19	2.47	1.06	2.51	1.07	2.42	1.08	2.41	1.07	2.48	1.06	2.50	1.07	2.43	1.07	2.45	1.05
20	2.34	1.04	2.45	1.05	2.45	1.04	2.41	1.05	2.39	1.03	2.43	1.04	2.43	1.04	2.38	1.03
21	2.27	1.01	2.36	1.01	2.31	0.99	2.32	1.01	2.36	1.01	2.41	1.01	2.36	1.01	2.38	1.00
22	2.28	0.98	2.30	0.98	2.30	0.98	2.30	0.99	2.36	0.97	2.22	0.96	2.34	0.98	2.29	0.98
23	2.08	0.94	2.21	0.93	2.22	0.92	2.27	0.95	2.34	0.94	2.18	0.92	2.21	0.95	2.27	0.94
24	**2.02**	0.93	2.04	0.88	2.10	0.89	2.11	0.91	2.26	0.93	2.21	0.92	2.32	0.92	2.19	0.92

$(E_A)_{min} = 2.02$ GPa, $(E_A)_{max} = 2.82$ GPa, $(\rho)_{min} = 0.88\ g \cdot cm^{-3}$, $(\rho)_{max} = 1.15\ g \cdot cm^{-3}$.
Nominal specimen thickness: 4 mm. Nominal specimen centerline distance: 14.22 mm. Nominal segment centerline distance: 12.7 mm.



(a) (b)

Figure 11 Variations of the Young's moduli versus the density ρ over a 101.6 × 304.8-mm area of a 4-mm-thick, 5%-density-reduction foam plaque. (a) Left modulus E_L and (b) average modulus E_A.



(a) (b)

Figure 12 Variations of the Young's moduli versus the density ρ over a 101.6 × 304.8-mm area of a 4-mm-thick, 15%-density-reduction foam plaque. (a) Left modulus E_L and (b) average modulus E_A.

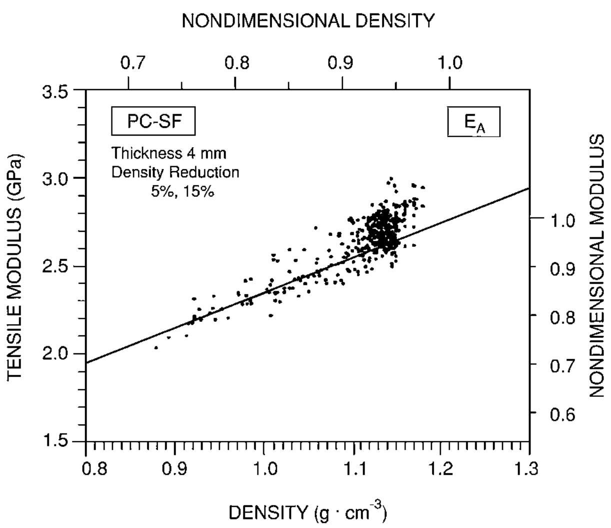


Figure 13 Variation of the average tensile modulus E_A versus the local density ρ over a 101.6 × 304.8-mm area of 4-mm-thick foam plaques, one for each nominal density reduction of 5 and 15%. (Superposition of data from Figs 11b and 12b.)

is much closer to $E_A = 2\rho + 0.35$ than is the data for the 5% density reduction material (Fig. 11b).

4.7. Density-modulus correlation for 4-mm-thick foam

The consolidated data on variations of the average tensile modulus E_A versus the local density ρ for nominal density reductions of 5 and 15%—from Figs 11b and 12c, respectively—are shown in Fig. 13. This figure shows that, just as in 6.35-mm-thick plaques, the average modulus appears to correlate linearly with the local density. However, the data for the 4-mm-thick material appears to lie above the solid line $E_A = 2\rho + 0.35$, which is a good fit for the 6.35-mm-thick material.

5. Flexural modulus

For 6.35-mm-thick plaques, measured values of the average flexural modulus, E_B, over 76.2-mm spans, at three locations—the 4–5, 12–13, and 20–21 segment

TABLE XII Variation of the average flow-direction flexural and tensile moduli, E_B and $\bar{E}_A$, in 6.35-mm-thick, polycarbonate structural foam plaques, for three nominal density reductions

Nominal density reduction (%)	Mean segment location number	Average flexural modulus E_B and average tensile modulus $\bar{E}_A$ (GPa)															
		Specimen number															
		1		2		3		4		5		6		7		8	
		E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$
5	4–5	2.75	2.64	2.75	2.62	2.75	2.65	2.66	2.62	2.65	2.56	2.65	2.65	2.68	2.63	2.74	**2.57**
	12–13	2.70	2.65	2.75	2.61	2.75	2.70	2.75	2.76	**2.62**	2.68	2.65	2.69	2.68	2.59	2.75	2.67
	20–21	2.75	2.66	**2.84**	2.65	2.80	**2.77**	2.75	2.63	2.75	2.62	2.71	2.75	2.75	2.65	2.75	2.74
15	4–5	2.66	**2.53**	2.55	2.40	2.62	2.40	2.55	2.36	**3.21**	2.46	2.66	2.46	2.68	2.44	2.65	2.47
	12–13	2.55	2.40	2.59	2.35	2.51	2.42	2.65	2.36	3.14	2.40	2.52	2.39	2.60	2.45	2.52	2.35
	20–21	2.38	2.32	2.38	2.24	2.38	2.36	2.29	2.31	3.03	2.27	2.38	2.24	2.38	2.31	2.38	2.21
25	4–5	2.62	2.32	2.51	2.38	2.52	2.35	2.38	2.35	2.58	2.34	2.56	2.38	**2.68**	2.35	2.61	**2.39**
	12–13	2.58	2.29	2.58	2.36	2.59	2.25	2.55	2.34	2.58	2.27	2.55	2.30	2.58	2.23	2.58	2.35
	20–21	2.38	2.20	2.38	2.27	**2.32**	**2.13**	2.38	2.22	2.38	2.21	2.38	2.17	2.38	2.15	2.38	2.16

TABLE XIII Variation of the average flow-direction flexural and tensile moduli, E_B and $\bar{E}_A$, in 4-mm-thick, polycarbonate structural foam plaques, for three nominal density reductions

Nominal density reduction (%)	Mean segment location number	Average flexural modulus E_B and average tensile modulus $\bar{E}_A$ (GPa)															
		Specimen number															
		1		2		3		4		5		6		7		8	
		E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$	E_B	$\bar{E}_A$
5	4–5	2.76	2.74	2.87	2.70	2.87	2.81	2.89	2.73	2.73	2.67	2.75	2.65	2.81	**2.86**	**2.91**	**2.86**
	12–13	2.79	2.76	2.68	2.76	2.63	2.83	2.60	2.78	2.62	2.72	2.68	2.79	2.49	2.84	2.49	2.79
	20–21	2.51	2.66	2.48	2.69	2.34	2.62	**2.26**	**2.57**	2.29	2.62	2.34	2.71	2.65	2.64	2.51	2.62
15	4–5	2.69	2.59	2.54	2.68	2.51	2.59	2.46	2.64	2.55	2.75	2.41	2.69	2.51	**2.78**	**2.75**	2.68
	12–13	2.62	2.54	2.41	2.58	2.46	2.59	2.62	2.64	2.52	2.66	2.60	2.66	2.58	2.68	2.67	2.65
	20–21	2.41	**2.32**	2.46	2.39	**2.25**	2.37	2.32	2.36	2.53	2.39	2.57	2.38	2.41	2.38	2.32	2.39

interfaces—along each of the eight specimens, are listed in Table XII, for nominal density reductions of 5, 15, and 25%. Also listed in this table are values of the arithmetic average of the tensile modulus, $\bar{E}_A$, over the 76.2-mm span used in the flexural tests. For each of the three density reductions, the maximum and minimum values of E_B and $\bar{E}_B$ have been highlighted in bold.

The plaque with a 5% nominal density reduction exhibits relatively small variations in E_B along any one specimen, with a maximum and a minimum of 2.84 and 2.62 GPa, respectively. The actual variability is much less than that corresponding to the extreme values. At these extremes, the ratio $E_B/\bar{E}_A$ has the values 1.07 and 0.98, respectively. In this plaque, the highest moduli are not necessarily in the area close to the gate. Surprisingly, in many cases, the highest moduli are in regions farthest from the gate.

In the 15% density reduction plaque, the maximum and minimum values of E_B are 3.21 and 2.29 GPa. At these extremes, the ratio $E_B/\bar{E}_A$ has the values 1.30 and 0.99, respectively. In the 25% density reduction plaque, the extreme values of E_B are 2.68 and 2.32 GPa. At these extremes, the ratio $E_B/\bar{E}_A$ has the values 1.14 and 1.09, respectively. At both these nominal density reductions, the moduli are highest near the gated end and generally lowest at the far end of the plaque. Also, in general, for equivalent locations, the moduli are higher in the 15% density reduction plaque.

Now the ratio $E_B/\bar{E}_A$ is a measure of how much material is in the outer layers of the material—this ratio being higher for thinner skins. The extreme values of 1.07 and 0.98 for this ratio, both of which are close to unity, indicate very thick skins with a small cellular core. Higher values of this ratio for the 15 and 25% density reduction plaques are consistent with them having thinner skins with thicker cellular cores.

Table XIII lists values of E_B and $\bar{E}_A$ for 4-mm-thick plaques with nominal density reductions of 5 and 15%. The maximum and minimum values of E_B for the 5% density reduction plaque are 2.91 and 2.26 GPa, respectively, and the corresponding values of the ratio $E_B/\bar{E}_A$ at these extremes are 1.02 and 0.88. For the higher (15%) density reduction plaque the extreme values of E_B are 2.75 and 2.25 GPa, and the corresponding value of $E_B/\bar{E}_A$ are 1.03 and 0.95. While the extreme values of E_B and the values of $E_B/\bar{E}_A$ at these extremes are on the same order as for the 6.35-mm-thick plaque for a density reduction of 5%, this is not true for a density reduction of 15%.

Table XIV lists values of $E_B/\bar{E}_A$ for both 6.35- and 4-mm-thick plaques for all the data in Tables XII and XIII. While this ratio is larger than unity at most

TABLE XIV Ratios $E_B/\bar{E}_A$ of average flexural modulus E_B to average tensile modulus $\bar{E}_A$ for 6.35- and 4-mm-thick polycarbonate structural foam plaques, for three nominal density reductions

Specimen thickness (mm)	Nominal density reduction (%)	Mean segment location number	$E_B/\bar{E}_A$							
			Specimen number							
			1	2	3	4	5	6	7	8
6.35	5	4–5	1.04	1.05	1.04	1.02	1.04	1.00	1.02	1.07
		12–13	1.02	1.05	1.02	1.00	0.98	0.99	1.03	1.03
		20–21	1.03	1.07	1.01	1.05	1.05	0.99	1.04	1.00
6.35	15	4–5	1.05	1.06	1.09	1.08	1.30	1.08	1.10	1.07
		12–13	1.06	1.10	1.04	1.12	1.31	1.05	1.06	1.07
		20–21	1.03	1.06	1.01	0.99	1.33	1.06	1.03	1.08
6.35	25	4–5	1.13	1.05	1.07	1.01	1.10	1.08	1.14	1.09
		12–13	1.13	1.09	1.15	1.09	1.14	1.11	1.16	1.10
		20–21	1.08	1.05	1.09	1.07	1.08	1.10	1.11	1.10
4	5	4–5	1.01	1.06	1.02	1.06	1.02	1.04	0.98	1.02
		12–13	1.01	0.97	0.93	0.94	0.96	0.96	0.88	0.89
		20–21	0.94	0.92	0.89	0.88	0.87	0.86	1.00	0.96
4	15	4–5	1.04	0.95	0.97	0.93	0.93	0.90	0.90	1.03
		12–13	1.03	0.93	0.95	0.99	0.95	0.98	0.96	1.01
		20–21	1.04	1.03	0.95	0.98	1.06	1.08	1.01	0.97

locations, implying that $E_B > \bar{E}_A$, there are regions where this ratio is less than unity. For foams with solid skins the expectation is that, locally, $E_B > \bar{E}_A$. This ratio being lower than unity could result from two approximations that have been made. First, the flexural moduli are averages measured over a span of 76.2 mm, and therefore could be lower than that corresponding to the local morphology. Second, the tensile modulus used in this ratio is an arithmetic mean over this span. A more representative mean, which would result in lower average tensile moduli and hence in higher values of this ratio, is the harmonic mean [12, 13].

6. Concluding remarks

While structural foam parts are normally specified in terms of a nominal density reduction, the data in this paper show that the actual local density in a part can be very different from that corresponding to the specified nominal density reduction, and may have a significant variation across the part. Also, even in parts of the same thickness, the local density at some location in a part with one nominal density reduction may be the same as at some other location in a second part with a different nominal density reduction. Furthermore, the local density can depend on the part thickness. In this paper, it has been shown that the local density reduction, as measured on 12.7×12.7-mm coupons from 101.6×304.8-mm regions of 6.35-mm-thick plaques, varies in the ranges of 3.3–8.3, 9.1–22.3, and 14.0–27.3% in plaques with nominal density reductions of 5, 15, and 25%, respectively. In 4-mm-thick plaques, the local density reduction lies in the ranges 2.5–19.8 and 5.0–27.3% in plaques with nominal density reductions of 5 and 15%, respectively. Since the local density is known to affect mechanical properties, these density variations imply in-plane mechanical property gradients in structural foam parts.

Because the in-plane bubble distribution and orientation are nonhomogeneous, the strains are nonhomogeneously distributed across any cross section of a tensile test bar. As a result, as shown in this paper, extensometers attached to either (thin) edge of a rectangular bar will not result in a meaningful value of the local modulus; the two moduli E_L and E_R so measured can be very different. Rather, $E_A = (E_L + E_R)/2$, which is expected to reflect the average strain across the face width, is an appropriate measure for the local tensile modulus.

The data in this paper show that, while the left and right tensile moduli, E_L and E_R, do not correlate with the local density, the average local tensile modulus exhibits a strong linear correlation with the local density ρ. The modulus-density data from 6.35-mm-plaques with nominal density reduction of 15 and 25%, which have regions with overlapping local density reductions, are indistinguishable in regions with common densities. Thus, for parts of the same thickness, the local elastic modulus is determined when the local density is known. The data from 4-mm-thick plaques with nominal density reductions of 5 and 15% also exhibit the same trends as those for the 6.35-mm-thick plaques. However, while the modulus-density correlation is still linear, the correlation for the thicker plaques appears to underestimate the local modulus at higher densities (lower density reductions). Now structural foams are normally used at nominal density reductions higher that 10%. In that range, the linear dependence of the average modulus on the local density is essentially the same for the 6.35-mm- and 4-mm-thick materials.

The experiments reported in this paper have established a strong linear correlation between the local modulus and the local density as measured in the plaque molding flow direction. Because the shear field can affect the bubble shape, the cellular morphology can be different in the flow and cross-flow directions. These differences could result in material anisotropy that needs to be characterized. However, the dependence of the cross-flow modulus on the local density is expected to be linear. A complete characterization of the elastic properties also requires a determination of the local Poisson's ratio and the local shear modulus. Also, the effect of the inherent variability in the modulus of these materials at all densities on the stiffness of parts needs to be quantified.

The empirical elastic-modulus density correlation provides the basis for a rational, finite-element-analysis (FEA) based mechanical design of structural foam parts. Once the local density distribution in the part geometry has been determined (predicted), the correlation determines the local elastic properties for the FEA. However, although some attempts have been made at predicting the local density of foams [14, 15], procedures for predicting the density distribution in parts of complex geometry are not presently available.

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